Map Projections

Miljenko Lapaine

Abstract. The paper is a review of the map projection theory and applications all over the world and in Croatia. First of all, cartography is shortly presented as a discipline dealing with foundation, production, promotion and studying of maps. Theory of map projections is a branch of cartography studying the ways of projecting the curved surface of the earth and other heavenly bodies into the plane. After a brief review of the history of map projections the paper gives the general outline of map projections and persons dealing with their theory on the territory of Croatia. In addition, the problem of the state map projection selection, which is one of the basic issues of the Croatian official cartography, has been explained.

Keywords: cartography, map projections, history, Croatia

1 Cartography and its Development

At its 10th General Conference held in Barcelona in 1995, The International Cartographic Association – ICA accepted the following definition with regard to the changes in technique and usage of cartography and maps and in accordance with the conclusion made by the Working Group on Cartographic Definitions:

A cartographer is a person dealing with cartography.
Cartography is a discipline dealing with foundation, production, promotion and studying of maps.
A map is a symbol model of geographic reality presenting the selected objects of properties, resulting from creative selection of its author, and used when the spatial relationships are of first class importance.

It is not easy at all to define map, cartography and cartographer. The above stated definitions are the result of the work that the Working Group to Define the Main Theoretical Issues on Cartography in the International Cartographic Association has been doing for many years. M. Lapaine participated at the meeting held in Bornemouth (8), and the detailed report about its work was given by its chairman Ch. Board (7, 9). Passing the above given resolution, i.e. accepting its definitions has not stopped the work on their improvement. It was noticed already in Barcelona in 1995 that the work should be continued, so the former Working Group to Define the Main Theoretical Issues in Cartography grew into the Commission on Theoretical Fields and Definitions in Cartography.

The term *cartography* is a compound made of two Greek words: χαρτηζ – meaning sheet of paper, charter, map and γραφω – meaning – write, draw. Cartography is an old art, old profession, but the word cartography was mentioned for the first time together with the words geography and chorography, engraved on a geodetic instrument from 1576 (67). As a term for scientific activity it was first used by a historian of geography, M. F. de Santarem in 1839.

Almost two millennia, cartography has been developing within the scope of geography as its integral part. The concept of cartography as science was presented for the first time under the title *geography* in the second century in the famous Ptolemy's work *Introduction into Geography.* In the Middle Ages, the regional direction in geography that was built on

Ptolemy's ideas, found its expression in the most important geographic works of that time, in big atlases from the 16th and 17th centuries that were collections of maps and comprehensive texts. The synonym for cartography was cosmography, and some cartographers of that time were called cosmographers. V. N. Tatishchev characterised the cartography of that time as *mathematical geography* and considered it one of the three constituent parts of geography. Mathematical geography and map projections were the fields of activity that Steinhauser (108) was dealing with for example in his textbook. Famous French cartographers from the 17th and 18th centuries, Nicolas Sanson, Guillaume Delisle and Jean-Baptiste Bourguignon d'Anville were called *royal cartographers.*

Geography of that time being of descriptive character and researching where things were situated, could not introduce into cartography new ideas or procedures to process cartographic material. Much more initiative came from mathematics that enriched mapping with analytical methods applied in the development of graticule, as well as with mathematical bases for topographic survey. At the same time the surveying of the earth's surface became the subject of geodesy as applied mathematical discipline. One of the geodetic disciplines, plane surveying or topography, developed the surveying methods for the production of detailed topographic maps so mapping was deduced to geometric registration of phenomena. It resulted in state cartographic works occupied above all with providing military with topographic maps. It weakened the connections between cartography and geography, but they were renewed later, on a new basis.

I would like to point out that among the authors of a booklet *Matching the Map Projection to the Need* published in 1991 by the Committee on Map Projections of the American Cartographic Association (20) there is only one professor of cartography (A. H. Robinson), and seven professors of geography (H. W. Castner, R. E. Dahlberg, P. P. Gilmartin, Mei-Ling Hsu, M. Monmonier, Ph. C. Muehrcke, J. M. Olson).

Wide apprehension of cartography has been created in its later development as an independent scientific discipline and in connection with the development and enlargement of high school cartographic education. I still believe that cartography cannot be developed only as independent discipline; its development depends essentially on other fields, first of all informatics, geography and geodesy.

The application of computers in cartography graws every day bigger and bigger, and many states have their cartographic data in digital form. Therefore, the standardisation of spatial database transfer is one of the most important tasks for cartography as profession. In order to understand the complete procedure of such a transfer, it is of greatest importance to understand the foundations of cartographic theory that it is based on. It includes the terms: real and virtual maps, deep and surface cartographic structures and cartographic data levels (77). These terms results from the development of analytical cartography, the area being the major agitator of the development in theoretical and mathematical cartography bases. Moellering has given a condensed presentation of analytical cartography (78).

The first two basic terms in this area are the terms about real and virtual maps. The 70-ties brought a lot of cartographic products, e.g. images on monitor screens and digital terrain models that have crossed the limitations of customary map definitions as permanent product on paper. In the leading article of the first issue of the magazine *American Cartographer* written by Morrison (79), the author notices the new problem and invites to a wider map definition. Moellering faced the same problem and accepted the challenge raised by Morrison. After a few years of research, the definition of real and virtual maps was suggested (76).

There are two decisive characteristics making distinction between conventional maps in real form of a hard copy, and virtual maps. The real map is a product that can be seen directly as cartographic image. Usual maps on a piece of paper, and images on the monitor screen can be seen in that way, but e.g. files with cartographic data cannot. They have to be transformed first into the form of direct visibility. The other decisive characteristic is whether the product can be touched. The classes of real and virtual maps obtained through the answers yes/no as related to the above mentioned two characteristics, are the real maps and virtual maps of the first, second and third type (76).

The conventional cartographic products as map sheets, atlases and globes which have hard touchable reality and are directly visible as cartographic images are called *real maps*. The other three classes missing one or both of the characteristics are called *virtual maps*. These three classes make it possible to widen the map definition, which reflects the development of modern cartography. Hence, virtual maps can contain the same information as real maps, and in the case of cartographic databases probably even more. Moellering (76) points out that also cartographic databases should be regarded as maps because they can contain information of real maps, and can also be transformed into them, if necessary.

Cartography can be divided in various ways. For example, we divide it into topographic and thematic according to the presentation object, to classical and automated according to production methods, to military and civilian according to its purpose, and furthermore to cadastral, plan, school, atlas etc. Borčić (12) divided cartography to general, mathematical and practical.

The division of cartography corresponding approximately to the process of producing and using maps is the following according to Lovrić (67): history of cartography, general cartography, mathematical cartography or theory of map projections, map design, map compilation, map publication, map use and map updating.

2 Theory of Map Projections

Theory of map projections is a branch of cartography studying the ways of projecting the curved surface of the earth and other heavenly bodies into the plane, and it is often called mathematical cartography. Many textbooks studying map projections are therefore titled *Mathematical cartography* (12, 26, 32, 38, 48, 49, 50, 75, 87, 88, 105, 117), and also Bugayevskiy and Snyder (15) in their latest book use the same term as a synonym for the theory of map projections. Today, however, the title mathematical cartography can have very wide meaning, so it should not be identified, according to Frančula, with the theory of map projections. Already Borčić (12) thought that also the map use, including map graphics as well should be listed into the mathematical cartography.

The goal of studying map projections is to create mathematical basis for the production of maps and solving the theoretical and practical tasks in cartography, geodesy, geography, astronomy, navigation and other related sciences.

In the creation of mathematical basis for a map one should construct a network of meridians and parallels in a selected projection, or of some other co-ordinate lines that have been used as a skeleton for entering other contents. The graphic way of constructing these graticules has completely met the demands of geography and cartography at the beginning. Later on, since the control points have been determined by means of triangulation, and original maps made on the basis of topographic surveys, it was necessary to make graticules more accurately, and it could be achieved by computing numerical values of co-ordinates of points along meridians and parallels in the plane of projection. We call this an analytical way, because the co-ordinates should be expressed by means of mathematical formulas, i.e. in the analytical form. It is therefore necessary to establish a functional connection among points on the surface of ellipsoid and in the projection plane for each map projection. This functional dependence is expressed with basic cartographic equations.

Basic cartographic equations enable computing and studying of distortions in map projections. Namely, while projecting the surface of an ellipsoid or sphere into the plane, it comes to distortions of lengths, surfaces and angles. On the basis of the size and disposition of distortions on maps, it is possible to compare projections and to select the most convenient ones. The selection of a projection is, however, influenced not only by distortions, but also by a larger number of other factors. Taking all that into consideration it is necessary to select the most convenient projection for the maps of certain area and certain purpose.

In the selected projection one should then construct the graticule. For this purpose the computer programs are produced today used for computing and drawing graticules for any part of the earth's sphere in any projection and scale. These programs enable also the drawing of other contents. The usage of computers and plotters in cartography has made it remarkably easier to search for and obtain new versions of already existing projections.

Fig. 1 Gilbert map projection

A part of map projection theory studying the ways of projecting certain points from the surface of the earth's ellipsoid by means of some geodetic methods is called *geodetic cartography.* In this part of the theory of map projections some other problems appearing in the conversion from ellipsoid into the plane are also studied, and these are: meridian convergence computation, scale computation, computing the reduction of lengths and directions, the first and the second geodetic problem and conversion of point co-ordinates while passing from one into another co-ordinate system in the same projection (12).

A map is not a simple drawing of the earth's surface, but its with symbols presented model obtained on the basis of certain mathematical laws. These laws presume indirect transfer from the physical earth's surface into its graphic presentation in the plane. First, we pass from the physical earth's surface onto the mathematical surface – rotational ellipsoid or sphere. This transfer is realised by means of orthogonal projection of physical surface points onto the mathematical surface by means of geodetic control network enabling correct geographic location and orientation of map contents within the frame of some co-ordinate network on the ellipsoid and then on the map. Afterwards follows the transfer from the rotational ellipsoid surface or sphere into the plane. These projections are called map projections and are the subject of the theory of map projections.

Curved surfaces of the ellipsoid and sphere cannot be flattened into the plane without distortions. However, the distribution of distortions and their amount can be determined if we know the projection law. In other words, the application of map projections enables numerical determination of deformation and their elimination from the data taken over from the maps.

Badly selected map projection can create bad consequences. One tries to believe in what is visible, and if basic geographic relationships as the forms, sizes, directions etc., are very much distorted, it is accepted as a fact if they can be seen as such on a map. This can lead to very wrong impressions, for example, that the route from Chicago to Rome is due east or from San Francisco to Tokyo is due west, or that North America is larger than Africa. Our mental maps, “databases” in our brain, are made only from what we see and from our experience. Since globes are not so frequent, the majority of mental images about the earth come from geographic maps. Very much distorted map that is often observed (as for example those in the background of television news) will soon look familiar and thus “correct”. It can become a deformed mental world map to some individual (19).

Among many map projections in use for world maps there are only a few of them in which the spherical shape of the earth is preserved to a greater extent. Among them, the Gilbert projection (Fig. 1) should be especially emphasized. It represents the world as people usually see it, from space and in the round. With its round shape it reminds of the globe, and at the same time it represents the whole earth's surface (58, 59, 61, 62).

In its efforts to express the problems it studies and to find the analytical solutions for them, the theory of map projections applies postulates of many mathematical disciplines, as for example plain and spherical trigonometry, differential geometry, differential and integral calculus, methods of numerical analysis etc. The theory of map projections takes the data about the shape and size of the earth and the co-ordinates of geodetic control points, and astronomical points over from geodesy and geodetic astronomy. It is also clearly shown that the theory of map projections is mutually connected with geography and other sciences that a great number of various maps are made for.

The basic tasks of the theory of map projections are the following according to Vakhrameyeva et al. (117), Bugayevskiy and Snyder (15), and Yang et al. (123):

- spreading of knowledge about map projections;
- development of the theory of map projections, primarily in the field of obtaining the best map projections, including the “best” and the “most ideal";
- researching various map projections, their essence, properties, mutual dependence and efficiency of their practical application;
- improvement of the existing map projections and their standardisation;
- exploring the role of map projections and their application in geoinformation systems;
- improvement of the methods for researching new map projections, especially algorithms and corresponding software accompanied by the improvement of computing techniques;
- processing of other mathematical map elements as scale, division into sheets etc., appearing in the publications on several sheets;
- testing and solving the tasks of mathematical character appearing in the compilation of maps (e.g. transformation of map projections by means of various tools, including computers);
- detailed work on the theory and methods of automation in the application of map projections;
- developing a projection method fit for solving the problems of planar and linear geometry in order to meet the needs of other sciences such as astronomy, crystallography, and geology;

- research space projections and the Landsat projection to meet the need of developing of space technology;
- working out digital data processing in map databases, spatial information positioning systems, and map projection transformation systems to meet the requirement of building different specific information systems and developing spatial information science.

3 History of Map Projections

Map projections have been developing parallel with the development of map production and cartography in general. The development of many sciences, technical achievements and the needs of everyday life have gradually initiated wider and wider demands for the production of various topographic and thematic maps in various scales and for various purposes, which requested continuous growth of map projections and improvement of mathematical basis of maps.

The beginnings of map projections date as far as two thousand years ago, originating from the time when the old Greek scientists introduced mathematical principles into the basis of projecting the earth and starry sky and started to apply the graticule. The works of Anaximander, Eratosthenes, Apolonious and Hipparchus played an important role in the development of cartography.

It is believed that Thales of Milet made the first map in some projection 600 years B.C. It was a map of the heavenly sphere in gnomonic projection. Stereographic and orthographic projections belong to the oldest projections and were used by the Greek astronomer and mathematician Hipparchus for the purpose of making maps of heavenly sphere about 150 B.C. Ever since there have been hundreds of map projections invented up to the present day.

In the 2nd century Ptolemy wrote a capital work *Geography* and included into it the description of map compilation and determination of the earth’s dimensions, as well as the construction of map projections. The period of Middle Ages in Europe was characterised by the so-called monastery maps reflecting religious image of the world.

Special development of cartography started during the renaissance – the period of great geographic discoveries. Accurate, reliable maps to be used for state government and military purposes, for the development of trade and maritime affairs. Such maps could be produced only through the application of mathematical basis and land survey results. The first to have appeared were topographic maps.

At the end of the 16th and 17th centuries, the compilation and publication of geographic atlases was a very significant event in further development and popularisation of cartography, which was carried out by the famous Dutch cartographers Ortelius and Mercator. Mercator was the first who ever applied conformal cylindrical projection used successfully for maritime navigation charts up to now.

In the production of world maps and the maps of larger territories, quadrangle projection and Apianus projection used to have large application in that time, and they were used as archetype for later more detailed pseudocylindrical projections. In the 17th century a new sinusoidal pseudocylindrical projection for the world map was suggested by the French cartographer N. Sanson.

Detailed work on the scientific basis of cartography and the beginnings of topographic studying of the earth, and as a result, further growth of accuracy and reliability of maps are regarded as the characteristics of the 18th century. A series of new projections suggested by R. Bonne, J. H. Lambert. J. L. de Lagrange, L. Euler and others, was introduced into the cartographic practice.

The renaissance came to an end with the introduction of elementary mathematical analysis into the development of map projections. Such analysis was applied especially to Mercator projection in which the rhumb lines were shown as straight lines, and to projections where all parallels are divided by meridians in true interspaces, regardless of the fact whether the parallels are circular arcs (Werner's projections) or straight lines (sinusoidal). Map projections become more complex: instead of those having graticule simply drawn because they consist of circular arcs and straight lines, there are such that are delineated by means of tables of trigonometric functions.

Murdoch (80) required consciously that the total area of the projected territory should be correct, but he did not insist on the constant local scale of the area. The first intentional preservation of area in each point was made by Lambert (57) by inventing cylindrical, azimuthal and conical equivalent projections. Halley proved geometrically the conformity of stereographic projection, and Lambert did it with differential calculus, and invented also three new conformal projections. Many map projections were created also in the 19th and 20th centuries, but the basic principals were laid until 1772, and especially in that year.

The most significant contribution of cartographers, geodesists and mathematicians of the 19th century to the theory of map projections was the establishment of firm mathematical principals. Lambert and Lagrange made important beginning in 1770, but especially Gauss and Tissot gave large contributions until 1880. The other researchers as e.g. Airy, Clarke, Schwartz and Pierce concentrated on more specific, but more complex tasks, and Germain, Gretschel and Craig gathered various works by other scientists into special monographs. New projections by Mollveide, Albers, Gall and others contain simpler application of mathematics, but still deserve the titles according to their inventors. Rapid development of the theory of map projections in this period is emphasised by a number of new projections and a number of published books and articles (101).

According to Frischauf (33), the beginning of the theory of projecting one surface onto another belongs to J. H. Lambert, who dealt with generally given problem of projecting a sphere and spheroid into the plane in his *Anmerkungen und Zusätze zur Entwerfung der Land- und Himmelscharten* (Remarks and Additions to the Establishment of Land and Sky Maps) in the third part of his *Beiträge zum Gebrauch der Mathematik und deren Anwendung (*Contributions to the Usage of Mathematics and its Applications, 1772). Lambert's colleague, J. L. Lagrange (56) was inspired by his separation of variables in the expression for the arc length differential in conformal projection and he solved the problem of conformal projecting the rotational surfaces publishing it in two treatises *Sur la construction des cartes géographiques* (About the Construction of Geographic Maps, Nouveaux Mémoires de l'Académie Royal de Berlin, 1779).

At the beginning of the 19th century military institutions started to produce topographic maps at large scales for which mathematical basis has special importance because the distances and directions were determined on these maps.

In 1822 the Royal Scientific Society in Copenhagen raised the question: to find general solution to the problem – a part of a given surface should be projected onto another given surface so that the image is similar to the origin in its smallest details. C. F. Gauß was awarded for the solution of this problem. His work was first published in Schumacher's *Astronomische Abhandlungen* in 1825 (34). Lagrange's and Gauß' treatise are in the volume No. 155 of Ostwald's classics of exact sciences. C. G. J. Jacobi (46) noticed in his *Vorlesungen über Dynamik* (Lectures on Dynamics), in the 28th lecture *Die kürzeste Linie auf dem dreiachsigen Ellipsoid. Das Problem der Kartenprojektion* (The shortest line on a three-axes ellipsoid. The problem of map projection), the following about the work by Gauß: "It contains Langrange's work that is only a little bit supplemented without being mentioned." Apart from that, Gauß gave only the examples of projecting rotational surfaces.

The projection of ellipsoid with various axes into the plane was processed in the above mentioned lectures by C. G. J. Jacobi. He gave the first announcement of such solution in *Monatsberichten der Akademie* and in the 19th volume of *Crelles Journal*. The complete solution together with other problems (projection of the rotational surface, cone, cylinder into the plane) was reported by L. Cohn on the basis of Jacobi's heritage in the 59th Volume of Crelles Journal under the title *Über die Abbildung eines ungleichachsigen Ellipsoides auf einer Ebene, bei welcher die kleinsten Teile ähnlich bleiben* (About the projection of the ellipsoid with unequal axes into the plane, whereby the smallest parts remain similar). This work contains also a very simple theory of conformal projection of one surface onto another. In the lectures about dynamics Jacobi explains: "The successful solving of the problem of conformal ellipsoid projection is achieved by means of adequate substitution and method leading to one partial differential equation. The noticed ellipsoid point will be determined by intersection of two curves of curvature". Disintegrating the ellipsoid surface into the elements limited by curves of curvature, Legendre already determined this surface (*Exercises du calcul integral,* Exercises in integral calculus, 1811) and found then corresponding variables by means of which the known projection problem could be solved. Ernst Schering gave complete solution of the problem on the basis of Jacobi's announcement in his work *Über die konforme Abbildung des Ellipsoides auf der Ebene* (About the conformal projection of ellipsoid into the plane) for which he was awarded in 1858 with the reward of the Faculty of Philosophy.

Surrounded by numerous works on cartography the majority of which is dealing with the methods of constructing graticules, I should by all means look back on the classical work, the corner stone of the theory of map projections by M. A. Tissot (111) *Mémoire sur la représantation des surfaces et les projections des cartes géographiques* (Treatise on surface presentation and geographic map projections, Paris 1881) the basic part of which, chapters from I to IV, appeared already between 1878 and 1880 in Nouvelles Annales des Mathématique, 2[e] série. According to K. Zöppritz, the work by Tissot was noticed in Germany and Austria as well, and E. Hammer (112) translated it into German under the title *Die Netzentwürfe geographischer Karten nebst Aufgaben über Abbildung beliebiger Flächen aufeinander* (Presentations of graticules of geographic maps including the problems on projecting one arbitrary surface onto another, Stuttgart 1887), in which many presentation of graticules and tables were added.

After 1900 about fifty monographs on map projections in about ten languages were published (101). There are a few in English and are very often quoted. In chronological order related to the first publication these are Hinks (41) *Map projections,* Deetz and Adams (24) *Elements of Map Projections,* Steers (107) *An Introduction to the Study of Map Projections,* Melluish (73), *An Introduction to the Mathematics of Map Projections,* Richards and Adler (92) *Map Projections for Geodesists, Cartographers and Geographers,* Maling (68) *Co-ordinate Systems and Map Projections,* Snyder (98) *Map Projections Used by the U. S. Geological Survey* and Snyder (100) *Map Projections: A Working Manual.* There is also the latest manual by Bugayevskiy and Snyder (15) *Map Projections – A Reference Manual.* One should mention also longer treatises with special topics about map projections by Adams (5, 6), Young (125), Thomas (110), Lee (66) and Snyder (99).

The following monographs in German should be mentioned: Maurer (72) *Ebene Kugelbilder,* Wagner (120) *Kartographische Netzentwürfe,* Merkel (74) *Grundzüge der Kartenprojektionslehre,* Hoschek (42) *Mathematische Grundlagen der Kartographie,* Kuntz (52) *Kartennetzentwurfslehre,* then a series of works by Bulgarian geodesist Hristow, published in Zeitschrift für Vermessungswesen, i.e. in books (43, 44) and the German translation from Czech Fiala (26) *Mathematische Kartographie.*

In French there are: Driencourt and Laborde (25) *Traité des projections des cartes géographiques* and Reignier (91) *Les systèms de projection et leurs applications.*

In the former Soviet Union many monographs were written, and the most prominent authors are: Kavrayskiy (48, 49, 50), Solov'ev (104, 105), Graur (38), Urmayev (116), Ginzburg and Salmanova (36), Meshcheryakov (75), Pavlov (87, 88), Vakhrameyeva, Bugayevskiy and Kazakova (117). The book by Bulgarian geodesist Hristow (45) was also published in Russian.

The application of map projections has a long history. Especially with the development of radio navigation and the technology of radio positioning, a series of new research topics has become the subject of map projection research. Map projections, as a means of establishing the relation between space and a plane, have been widely used to solve some geometric problems of spherical geometry, astronomy, crystallography, and geology in graphical form. The application of Landsat to mapping introduced completely new concepts for map projections. The time has now become a parameter in mapping. This is quite different from conventional static mapping in which the relations among the earth's shape, perspective center and projection plane are fixed. The entirely new research topic of studying a projection suitable for satellite mapping confronts map projection science. In recent years, the electronic computers, especially the personal computers, have been widely applied to all aspects of map projections and have thoroughly changed the look of map projection science. Examples are the applications of computers to the calculation of coordinates, to the automatic creation of the mathematical foundation of maps, and to the automatic plotting of thematic mathematical elements on a map. Computer-aided map projection transformation is even more of a leap for cartography. To meet the need of computer cartography, it is a pressing task to study the theory and methods of map projection transformation, to study topographic data processing, spatial information positioning, and transformation in information systems (123).

Geoinformation systems (GIS) enable today that some problems that have been so far solved only on geographic maps (various cartometric problems) are now solved directly from the databases. On the basis of that fact we could come to a conclusion that geoinformation systems lessen the significance of geographic maps. If we look upon it from that point of view, then it really is true, but on the other hand geographic maps are very important for every GIS. They have very important role in creation of databases, but also as one of the forms in presenting the output data (96).

Since each geographic map is made in a certain map projection, one can come to a conclusion that map projections are especially important in creating geoinformation systems. In the creation of national digital bases of geodetic, topographic and cartographic data that must make the foundations of each GIS being prepared for the territory of the entire state, the method of digitising the existing maps is very important (106, 109). Therefore, the majority of GIS software contain also the module for digitising. In the application of this module it is necessary to be familiar with the map projection of the origin and projection constants (e.g. geographic longitude of the mean meridian or the latitude of the standard parallel, and linear scales along them).

These data are indispensable in order to transform the co-ordinates from local digitizer system into the system of source map projection and then by means of inverse equations of map projections into the system of geographic co-ordinates. It would thus be possible for GIS software as one of the presentation forms for output data to offer geographic map in one of a great number of the most important map projections. In order to draw such a map, it is necessary to calculate rectangular co-ordinates x, y in the selected map projection from the geographic co-ordinates.

The production of software for any GIS requires thus the principal and inverse equations for a larger number of map projections. Hence, the computer aided method in the map production and first of all geoinformation systems have not reduced, but increased the importance of map projections. More than 1000 works on map projections published after

1960 and registered in the bibliography of Snyder and Steward (102) prove that this statement is correct.

This is the reason why special attention has been paid to map projections within the scope of the project *Cartography and Geoinformation Systems*, which was going on at the Institute for Cartography at the Faculty of Geodesy, University of Zagreb from 1991 to 1996.

4 Several Characteristics of Development of Modern Map Projection Science

With the advance of science and technology, there have been breakthroughs in the field of classical research objects and methods of map projection. Several characteristics of this development can be described as follows (123).

- The first characteristic is that research on the theory and methods of map projection is being deepened and combined with many special fields.

Some references to papers published after the 1980s can be the examples to illustrate these characteristics. In the paper *On the classification of map projections* (65) the author gave his new views on the map projections classification. He summed up projections as belonging to three types – elliptic projections, parabolic projections and hyperbolic projections – according to whether or not the equilength direction of arbitrary points in the projection area is the sole one. Furthermore, the study of the azimuthal projections has been deepened in recent years. As an example, the paper *The perspective azimuthal projection under variable view points* (122) combined perspective azimuthal projections with non-perspective azimuthal projections and unified the treatment of perspective azimuthal projection systems using variable view points.

Research into the theory and method of conformal projections has also been further developed. The monograph *Map Projections: a Reference Manual* by Bugayevskiy and Snyder (15) summed up and systematized the conditions of optimum and ideal conformal projection exploration methods and developed the method of the Chebyshev projections. The monograph *The Theory of a Single Space Photograph* by Bugayevskiy and Portnov (14) is an example of the integration of two research fields: cartography and space photography. With the advance of modern science and technology, it is increasingly important that scientists in different scientific and research fields be united to overcome difficulties and develop new technology arising from the sinergy of their efforts.

- The second characteristic is that new research and application areas are continuously being developed.

The study of map projection transformation not only opens up new research areas for map projections but also, with the computer-aided creation of the map's mathematical foundation and the construction of map projection transformation software systems, develops new application areas for map projections.

The traditional map's mathematical basis is the geographic network but this incompletely meets the needs of newer practical uses. This is because the advance of modern science and technology needs increased spatial positioning data, including such information as the position of points, or distance, azimuth, and track data. It can be said that the development of thematic mathematical elements for a map enlarges the application area of map projection science.

- The third characteristic is that the continuous presentation of new concepts promotes the advance of map projection science.

To meet the needs of developing the technology of remote sensing from space, a proposal was put forward in the 1970s by the US Geological Survey, suggesting the study of a space projection (18). The mathematical model for a space oblique Mercator projection was derived by J. P. Snyder in the late 1970s and was used for satellite series mapping. The theory and method for a space conformal projection was presented in the beginning of 1990s by a Chinese cartographer Cheng (16). With the emergence of space projections, there have been breakthroughs in the field of classical research on map projections. The functional relation between a spatial point (in three dimensions) and a plane point has been extended to a functional relation between a spatial point (in four dimensions) with the parameters of a spatial point's position in time and on a plane.

Variable scale projection is a newly developed and widely used map projection in recent years. There is a classical objective for a general map, namely to choose a projection that has limited deformation. But with a variable scale projection, according to the map's uses and requirements we can choose a projection that retains only the map graphic's topological relations. Chinese cartographers have been engaged in research into variable scale projection and have achieved many results. For example, the paper *Methods for Variable Scale Map Projections* (121) introduced many methods for variable scale projections. How the variable scale map projections work can be seen from Fig. 2 and 3.

- The fourth characteristic is that map projection and its transformation, as a spatial information positioning model, has become an important component of space information science.

Map projection transformation is a new field of research in the field of map projections. It involves exploring the theory and methods of transformation of the coordinates of a point from one type of projection to another.

In geodetic and topographic surveying, coordinate transformation between different systems, i.e. computation of coordinate zone transformations, is always needed. For a long time, many computational methods have been presented all over the world. In conventional cartography, in order to combine all the basic characteristics onto the graticule when compiling a new map, map projection transformation is implemented using photography in the transferring of coordinate data and rectification. By using these methods the coordinates of only a few points need to be rigorously transformed. For most points the subsequent transformation is approximate. This is characterized by the fact that we don't need to create a rigorous mathematical relationship for transformation of all points between two types of projections. The rigorous mathematical relations to compute a zone transformation are usually limited to a transformation between different coordinate systems for the same type of projection. For this reason, it belongs to a special case of map projection transformation, and is one of the fields of research in map projection transformation. The similar methods can also be applied to study maps whose projection is not given, as on ancient maps (113, 115) or to geometry which is implicit in 'mental maps' (114). By studying map projection transformation we can also develop some new map projections.

No activities of mankind can take place without geographical space. With spatial information rich digital maps and remote sensing images as geographical foundations have been combined with positioning models (map projections and their transformations) to form a spatial information positioning system (SIPS) (124). This is the carrier of all spatial information and is the foundation of positioning. The result is spatial information with a united geographical and plane coordinate system. Today, spatial information graphical positioning systems based on digital maps have permeated all aspects of GIS and have become an important component thereof. Space information science is a comprehensive discipline while the spatial information graphic positioning system is one of its important theoretical bases and an important component (123).

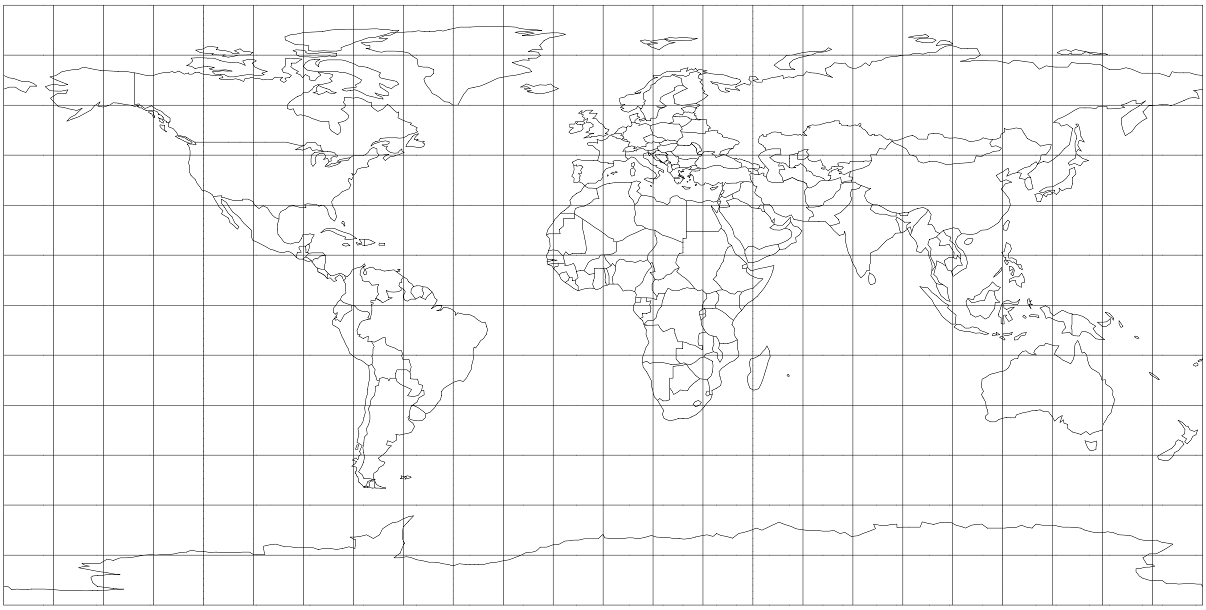

Fig. 2 Equidistant cylindrical map projection

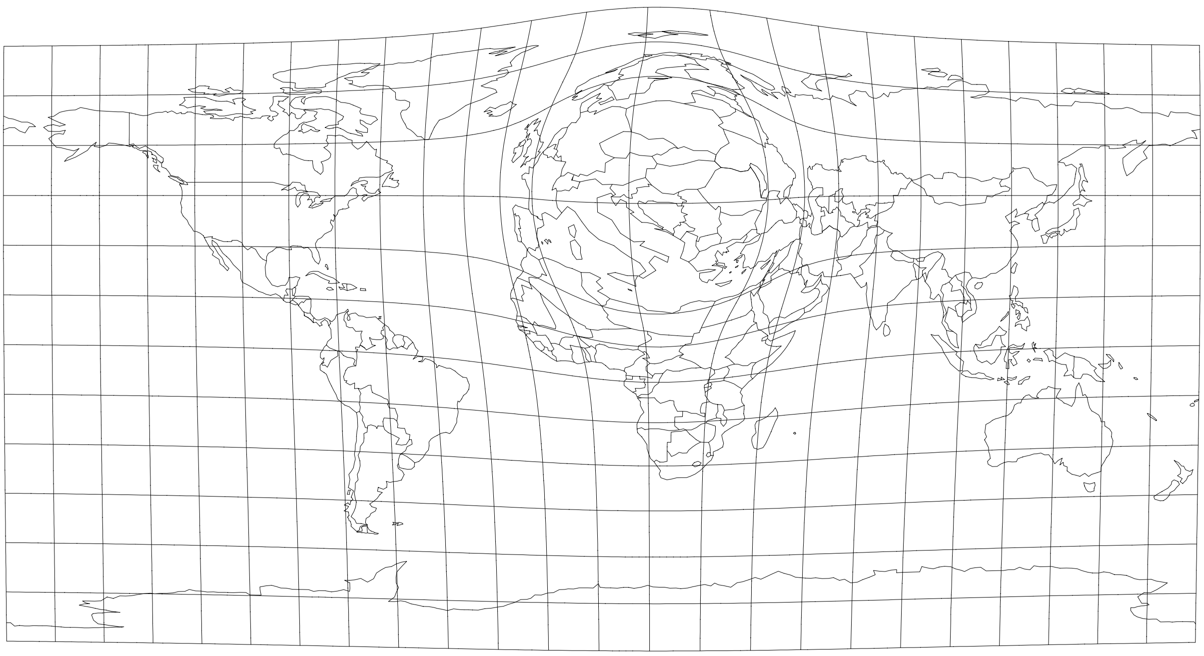

Fig. 3 Variable-scale map projection

5 Map Projections in Croatia

For the time being there is no comprehensive history of Croatian cartography, but there is a series of works on it, among which *Descriptio Croatiae* by Marković (70) is certainly one of the most valuable. However, as the author himself says in the Introduction, this work is not intended to be the history of cartography in Croatian countries, but to be cultural and historical presentation of main cartographic documents from which the course of getting to know Croatian countries from the oldest time up to the end of the 19th century can be followed.

Plenty of cartographic material about Istria and the entire Adriatic coast has been processed by Lago and Rossit (54), Kozličić (51), and Lago (55). The historical survey of cartography has been given by A. Pandžić in her catalogue on old maps and atlases in the Historical Museum of Croatia (84), and five centuries of geographic maps of Croatia are presented in the catalogue of the exhibition (85) bearing the same name and held in Zagreb in 1988, and about Croatia and its borders on maps from the 12th century up to now (86) in the catalogue of the exhibition held in 1992. The above mentioned sources contain important lists of literature that can be used in further researches.

If we stay within the frame of map projections, then one should say that there is still no comprehensive historical presentation of map projections and persons dealing with their theory on the territory of Croatia. I shall therefore give the general outline of so far acquired, my own insight that might be useful and direct further research in this area.

Croatian leading persons testify about the scientific tradition of Croats dating as far as the 12th century. Some of them gave such contributions to science that their achievements present new stages in the development of science. These are e.g. Herman Dalmatin in the 12th century and Josip Ruđer Bošković in the 18th century who can be regarded as forerunners of Croatian geodesy and cartography.

During the 19th and at the beginning of 20th century, the cadastral surveys were made at the territory of Croatia and plans were made in several co-ordinate systems. All these systems are called old systems. Since the plans made in these systems are still used, there is a need appearing to establish the connection between these systems and the system of Gauss-Krüger projection. The old survey was made by the experts led by the institutions with residence in administrative centres of the government of that time (Vienna, Budapest) so that very few data have been kept until the present days, out of which we could see how these works were done. The old co-ordinate systems at the territory of Croatia are: Kloštar-Ivanić, Budapest, Vienna, and Krim as well as one introduced by the oblique conformal cylindrical projection. The insight into the development of the old systems and their characteristics is given in a few articles (11, 21, 27, 97), and especially important is the book by J. Marek (69) *Technische Anleitung zur Ausführung der trigonometrischen Operationen des Katasters* from 1875.

Borčić and Frančula have worked with collaborators on the issue *To Determine the Elements of Mutual Transformation between Projections and Co-ordinate Systems, and the New Land Survey on theTerritory of the SRH* in the period from 1962-66. The basic results of the extensive research were published in the work *The Old Co-ordinate Systems on the Territory of the SR Croatia and their Transformation into the Systems of Gauss-Krüger Projection* (13). Later on, Frančula worked out especially the double oblique conformal cylindrical projection (28).

Although very important research has been carried out on map projections in the area of Croatia in the 19th and at the beginning of the 20th century, it seems that there is still a lot of material to study. For example, these are *Mitteilungen des k. k. Militär-Geographischen*

Institutes that used to be published in Vienna, and in which relatively extensive articles by H. Hartl about map projections have been published (39, 40).

David Segen (1859-1927) won his doctor's degree at the Faculty of Philosophy in Zagreb in 1889 as the first doctor in the field of mathematics at the University of Zagreb. He held many lectures, with the *Perspektivne mrežotine kartografijske* (Graticules in perspective projections), among them. He wrote a detailed article under the title *O crtanju mreža za geografijske karte* (About drawing graticules for geographic maps) that was published in Izviešće o Kr. velikoj realci u Osieku at the end of the school year 1880/81 (94). He also published in Nastavni vjesnik an article titled *Osnove reljef perspektive* (The elements of relief perspective) in 1893 (95).

Marije Kiseljak (1883-1947) lectured *Mathematical theory of cartography* (1922-23) and *Cartography* (1923-25) at the Geodetic and Engineering Department of the Technical High School of that time in Zagreb.

Vladimir Vranić worked from November 1945 at the Geodetic Department of the Technical Faculty as professor of *Mathematical cartography.* His work *O izvođenju formula sferne trigonometrije s pomoću stereografske projekcije* (About the derivation of formulas of spherical trigonometry by means of stereographic projection) is very important for the history of map projections, and it was translated into German *Über die Ableitung der Formeln der sphärischen Trigonometrie mit Hilfe der stereographischen Projection* (119).

Željko Marković (1889-1947) lectured mathematics at the Technical Faculty in Zagreb for many years. He did not lecture cartography, but it is important that he worked out in details in his textbooks *Higher Mathematics* the conformal mappings with a special reference to conformal mappings of sphere and ellipsoid in cartography (71).

For the history of map projections it is recommendable to remember that all generations of students of mathematics who attended the course of lectures *Complex Analysis* got familiar with stereographic projection and conformal mappings. Namely, if we want to make the space of complex number compact, we add an infinitely far away element to it, and it can be done and clearly represented by means of stereographic projection. Conformal mappings are an integral part of the Complex Analysis, it is only the question whether we should approach them abstractly, as it is done e.g. by Kurepa and Kraljević (53), or in a concrete way with the examples from cartography (71).

Let us mention as well that Vilko Niče (1902-1987) in his *Perspektiva* (Perspective) that has had a few publications since 1953, deals with the perspective projection of a globe among other things, an in the chapter on central projection applications he works out the stereographic and gnomonic projection (82).

Geodesist Jaroslav Rajtr writes about distortions of map projections in his article *Općenite zasade kartografije* (General Principles of Cartography) published in 1914 in *Vijesnik*, the paper of the Association of Civilian Technicians of the Kingdom of Croatia and Slavonija (89).

Tomo Jakić (1879-1966) from Požega was a professor of mathematics and physics at high schools in Požega and Zagreb. In *Nastavni vjesnik*, being published in 1916 in Zagreb for 24 years already, he published a very extensive article titled *Crtanje mreža za geografske karte* (Drawing the Graticules for Geographic Maps). The article came out in four instalments on altogether 52 pages (47).

Artur Franović Gavazzi (1861-1944) is the most prominent physical geographer in Croatia at the turnover from 19th to 20th century. His booklet containing 66 pages titled *Kartografijske projekcije* (Map Projections) is very important for the history of map projections, and it was published by Academic Geographic Club in Zagreb along with other manuscripts of his lectures at the University (35).

In the 20th century there have been a few of our geodetic experts who did not have map projections as their major field of interest but have still left written evidence about this part of cartography as well. I am giving them in chronological order: V. Filkuka (27), N. P. Abakumov et al. (3), N. Čubranić (21, 22, 23), E. Adamik (4), L. Randić (90), M. Bolt (10), Z. Narobe (81), R. Solarić (103), and M. Cigić (17).

The Institute for Cartography at the Faculty of Geodesy, University of Zagreb was founded by the Act of the University of Zagreb on 22nd May 1956 at the Geodetic Department of the Technical University of that time. The foundation of the Institute for Cartography was proceeded by the foundation of the *Department for Cartography* in 1951 after Branko Borčić had been selected an associate professor, i.e. gradual forming of the *Cartographic Laboratory* initiated by Ivan Kreiziger in 1948.

Remarkable teaching activitiy existed in the field of cartography even before foundation of the Department for Cartography. However, since its foundation, and especially after the establishment of the Institute for Cartography, a significant scientific and professional activity developed. The library of the Institute for Cartography is very rich and has about 5000 items. The Institute receives regularly 27 journals. The collection of maps contains about 8000 maps and 170 atlases.

At the Faculty of Geodesy, University of Zagreb there has been a continuation in teaching about map projections and geodetic drawing lasting for many years. Map projections are lectured by the most prominent professors: Dr. Marije Kiseljak, Dr. Vladimir Vranić, Dr. Antal Fasching, Nikolaj Abakumov, Stjepan Horvat, Dr. Franjo Braum, Mato Janković, Dr. Branko Borčić and at present Dr. Nedjeljko Frančula and Dr. Miljenko Lapaine.

Antal (Antun, Anton) Fasching (1879-1931) was one of the most educated Hungarian geodesists. He wrote about 70 works, taught Higher Geodesy, *Cartography, State Survey* and Photogrammetry in Zagreb in the period from 1923 till 1928. Apart from his teaching activity, his scientific work is very important for the theory of map projections, first of all his work on solving the issues connected with the selection of state projection.

Nikolaj Pavlovič Abakumov (1882-1965) was elected a full professor at the Geodetic Department of the Technical Faculty in Zagreb in 1927, and he taught until the end of 1950 *State Survey,* Higher Geodesy, Spherical Astronomy, Practical Astronomy, Applied Astronomy, Positional Astronomy, Photogrammetry, *Cartography, Mathematical Cartography, Applied Cartography* and Geophysics. I would like to mention that Borčić when writting his textbook quoted in the reference list the lectures of professor Abakumov at the Technical Faculty in Zagreb.

Stjepan Horvat (1895-1985) lectured Geodetic Cartography within the frame of the course of lectures *State Survey*, (Državna izmjera, Premjeravanje države). It was not very much known about him until recently, because after the II World War he was forced to emigrate. His works are important for the history of map projections because of their contents and range. If we would count only his works from the area of geodesy and cartography, we would obtain a number greater than hundred with altogether more than 2000 pages.

Branko Borčić (1908-1977) won his doctor's degree in 1964 at the Faculty for Architecture, Civil Engineering and Geodesy in Ljubljana with the thesis *Basis of the World Map at the Scale of 1:1 000 000.* From 1951 he was an associate professor at the Technical Faculty in Zagreb where he habilitated with the thesis *Contribution to the Solution of Co-ordinate Transformation between Neighbouring Co-ordinate Systems in Gauss-Krüger Projection.* Since 1960 he was a full professor at the Faculty of Architecture, Civil Engineering and Geodesy in Zagreb. He wrote about issues of mathematical and geodetic cartography and published in Geodetski glasnik, Geodetska služba, Geodetski list, and Građevinar. His textbooks about map projections and Gauss-Krüger projection were the only manuals of this type for years.

6 Selection of the Croatian Official Projection

One of the basic issues of the official cartography is the *selection of the state projection,* i.e. *the selection of the state co-ordinate system.* Such a selection was increasingly important at the beginning of the century, and then at the moment of creating the Independent State of Croatia (Nezavisna država Hrvatska), and is so at the present time.

The issue of selecting the projection and coordinate systems was inititated in 1921. The director of cadastre at that time appointed a well-known geodetic scientist, professor A. Fasching who came from Budapest, to solve this problem. The complexity of the job can be seen from the references (93).

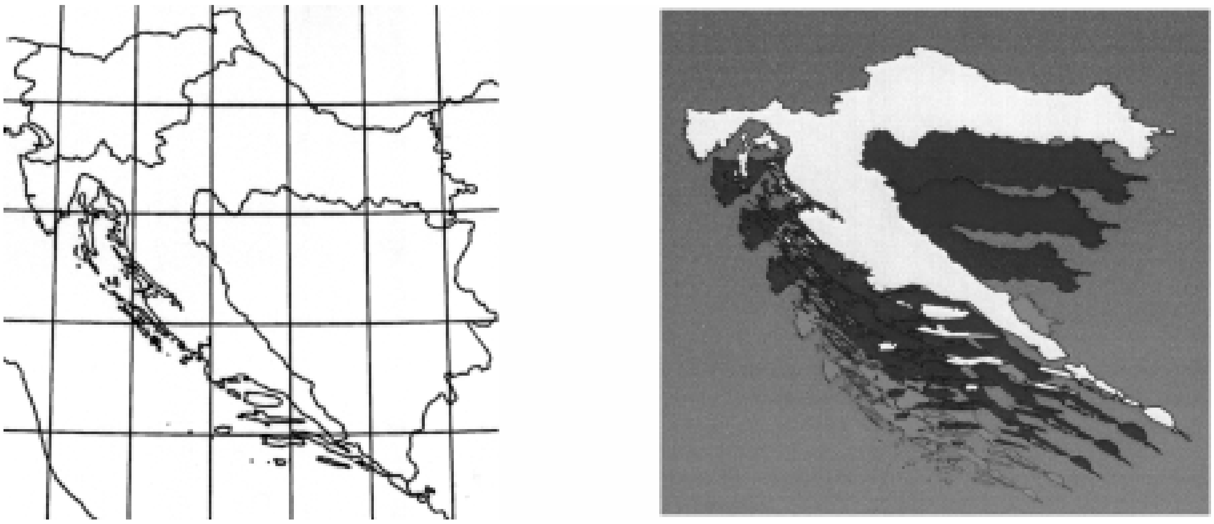

Fig. 4 Correct and distorted presentation of Croatia

As soon as one starts to talk about map projections, all experts unintentionally remember the famous saying by Jordan: "*No institution goes so deeply into the essence of the entire land survey and mapping of a country, as the selection of the projection and co-ordinate system. If a mistake is made here, it takes revenge on many generations.*" According to Abakumov (2) we felt in Croatia these prophetic words to full extent on our skins. From former Austro-Hungarian Monarchy we have inherited a few co-ordinate systems. Every expert working in the field of geodesy knows well what kind of troubles and difficulties are caused still today by the co-ordinates of the control points at our territory. Abakumov wrote in 1942 about the issue of selecting the most appropriate projection for the Independent State of Croatia and considered two variants of Gauss-Krüger projection, their advantages and disadvantages.

Gauss-Krüger projection is widely applied in geodetic practice (29). In many European countries this projection has been adopted as official state projection. It is well known that it was selected in 1924 for the territory of former Yugoslavia and that the Cartesian co-ordinates of points of state triangulation have been so far calculated and presented in this projection. Regarding new circumstances, and above all the shape of the Republic of Croatia and its spreading, it is necessary to be familiar with its most appropriate projections.

The scientific and professional project *Selection of State Map Projection* was suggested to the State Geodetic Administration in 1994, 1995, 1997 and 1998. Here are some examples showing the necessity of making competent judgement about the selection of map projection at the territory of Croatia.

1) Today more and more people are dealing with the production of maps by using computers. The interest in maps and cartography is undoubtedly growing. There were 16 reports at the CAD Forum'93 within the frame of the section on spatial information systems, and the majority of them were accompanied by the presentation of various maps. In one of these works (118) the transformation of the entire contents into the 5th system of Gauss-Krüger projection has been applied for the purpose of unified cartographic presentation, and the contents referred to the entire Croatia. It can be shown (60) that the application of the 5th system of Gauss-Krüger projection to the entire Croatia has no justification. On one hand we have obvious lack of symmetry in presentation, and on the other hand the distortions in the eastern part of the state are unnecessarily too large.
2) In Geodetski list No. 1 from 1997 there was an article published about the selection of map projection for the maps at small scales (30). In the introduction the authors point out the main reason that initiated the article: "The indirect motive are very often more and more frequent distorted presentations of Croatia as the result of insufficient knowledge in cartography, especially the knowledge about map projections and their selection for the maps of certain areas." The article has 10 pages, extensive reference list and two illustrations. How can we then explain the fact that the cover pages of the third number of Geodetski list from 1998 contain deliberately distorted map of Croatia.
3) It is not alarming if for example an architect or forester make a map of Croatia in inadequate map projection, or if a lawyer has some thoughts of his own about Gauss-Krüger projection, but something like that should not be accepted by the qualified engineer of geodesy. For example, if we say that Gauss-Krüger projection has been applied with the following parameters:

   - six degrees zone with the mean meridian of 16°30'
   - scale reduction coefficient at the meridian of contact is 0.9997
   - additional constant is 2 500 000,

   and the title contains State Geodetic Administration (37, 83), then it is obvious that the State Geodetic Administration did not pay enough attention to the map projections. There is namely no need to point out the six degrees zone because there is only one zone being approximately like that. Furthermore, it is not the matter of meridian of contact but of the central meridian of the area to be projected. Moreover, once in the time of logarithmic tables it had some sense to add some constant to the rectangular co-ordinates in Gauss-Krüger projection in order to avoid the computation with negative numbers, but today, in the time of computers or cybernetic era it looks rather exotic. Finally, it is not correct to credit all that to Prof. Frančula (83), because Frančula has never formulated his suggestion of map projection for the small-scale maps of Croatia in such a way. In order to illustrate the inability of recognising the problem of map projection I shall use the copy of the distorted map of Croatia to be found on the cover pages of at least two written materials made by the State Geodetic Administration, see Fig. 4. It is very instructive to see the figure at the page 162 in the Proceedings of the symposium "State Geodetic Bases and Land Information Systems" held in 1999 in Opatija (63). The figure shows the distortion that was caused by unauthorised intervention of the technical editor (spreading of the picture in only one direction), which made the text under the picture lose its meaning, and the efforts of the author to indicate the unnecessary deformed presentations have failed.

Some other examples of unnecessary distorted representations of the Croatian territory can be found in (30, 31).

At the end of this paper I would like to report that the State Geodetic Administration of the Republic of Croatia recognised the problem of mathematical basis for official cartography and ordered from the Faculty of Geodesy, University of Zagreb the elaboration of the project "Proposal for Official Map Projections of the Republic of Croatia" at the end of 1999 (64).

References

(1) Abakumov, N. (1942): Gauss-Krüger-ova projekcija u primjeni na područje Nezavisne Države Hrvatske, Hrvatska državna izmjera, br. 5, 70-74

(2) Abakumov, N. (1946): Koordinacija svjetskih geodetsko-kartografskih radova, Geodetski glasnik br. 2, 36-38.

(3) Abakumov, N., Dražić, M. P., Svečnikov, N., Sviščev, I. (1928, 1929): Projekcija novog katastarskog premera u Kraljevini SHS. Geometarski glasnik 1928, 3, 53-61; 4, 101-109; 5, 165-170; 1929, 1, 16-25; 2, 80-87; 3, 130-135.

(4) Adamik, E. (1949): Redukcija smjera i dužine u Gauss-Krügerovoj projekciji, Geodetski list, 46-50.

(5) Adams, O. S. (1919): A Study of Map Projections in General. U. S. Coast and Geodetic Survey Special Publication 57. Reprinted 1934.

(6) Adams, O. S. (1949): Latitude Developments Connected with Geodesy and Cartography. Special Publication No. 67, Department of Commerce, U. S. Coast and Geodetic Survey, Washington.

(7) Board, Ch. (1989): Report of Working Group on Cartographic Definitions. ICA Newsletter 14, 14-15.

(8) Board, Ch. (1991): Working Group on Cartographic Definitions. The Cartographic Journal, Vol. 28, No. 2, 249-250.

(9) Board, Ch. (1992): Report of the Working Group on Cartographic Definitions. The Cartographic Journal, Vol. 29, No. 1, 65-69.

(10) Bolt, M. (1974): Transformacija koordinata između susjednih koordinatnih sistema Gauss-Krügerove projekcije, Geodetski list, 18-22.

(11) Borčić, B. (1954): Kloštar-Ivanićki sistem. Geodetski list 1-4, 41-62.

(12) Borčić, B. (1955): Matematička kartografija. Tehnička knjiga, Zagreb, 408 str.

(13) Borčić, B., Frančula, N. (1969): Stari koordinatni sustavi na području SR Hrvatske i njihova transformacija u sustave Gauss-Krügerove projekcije. Geodetski fakultet Sveučilišta u Zagrebu.

(14) Bugayevskiy, L. M., Portnov, A. M. (1984): The Theory of Single Space Photographs, Nedra, Moscow.

(15) Bugayevskiy, L. M., Snyder, J. P. (1995): Map Projections – A Reference Manual, Taylor & Francis, London.

(16) Cheng, Yang (1992): On conformal projection maintaining a desired curve on the ellipsoid without distortion, ASPRS/ACSM, Summer Convention, Washington, Technical papers, 3, 294-303.

(17) Cigić, M. (1981): Transformacija koordinata iz sustava stare izmjere u sustave Gauss-Krügerove projekcije i obrnuto, primjenom džepnog računala, Geodetski list, 213-220.

(18) Colvocoresses, A. P. (1974): Space Oblique Mercator, Photogrammetric Engineering, 40, 921-926.

(19) Committee on Map Projections (1986): Which Map is the Best? Projections for World Maps. Special Publication No. 1 of the American Cartographic Association, American Congress on Surveying and Mapping, Falls Church, Virginia.
(20) Committee on Map Projections (1991): Matching the Map, Projection to the Need. Special Publication No. 3 of the American Cartographic Association, American Congress on Surveying and Mapping, Bethesda, Maryland.
(21) Čubranić, N. (1947): Projekcioni sistemi u Hrvatskoj. Geodetski list 1-2, 3-19.
(22) Čubranić, N. (1948): Tablica za redukciju dužina kod Gauss-Krügerove projekcije, Geodetski list, 14-16.
(23) Čubranić, N. (1954, 1974): Viša geodezija, I dio. Sveučilište u Zagrebu, Zagreb.
(24) Deetz, C. H., Adams, O. S. (1934): Elements of Map Projection with Application to Map and Chart Construction. U. S. Coast and Geodetic Survey Special Publication 68, Washington.
(25) Driencourt, L., Laborde, J. (1932): Traité des projections des cartes géographiques à l'usage des cartographes et des géodésiens. Hermann et Cie, Paris.
(26) Fiala, F. (1957): Mathematische Kartographie. VEB Verlag Technik, Berlin.
(27) Filkuka, V. (1919): Projekcije zemaljske izmjere u Hrvatskoj i Slavoniji, Glasilo geometara, br. 1 i 2, 2-5; br. 4, 5 i 6, 49-54; br. 7 i 8, 81-90.
(28) Frančula, N. (1972): Dvostruka kosa konformna cilindrična projekcija. Zbornik radova, publikacija br. 9, Geodetski fakultet Sveučilišta u Zagrebu, 85 str.
(29) Frančula, N. (1974): Izbor projekcije, Geodetski fakultet Sveučilišta u Zagrebu.
(30) Frančula, N., Lapaine, M., Vučetić, N. (1997): Izbor kartografske projekcije za karte sitnih mjerila, Geodetski list 1, 43-52.
(31) Frančula, N., Lapaine, M., Vučetić, N. (2000): Auswahl einer geeigneten kartographischen Abbildung für kleinmaßstäbige Karten, in: Per Aspera ad Astra, Festschrift für Fritz Kelnhofer zum 60. Geburtstag (Hrsg. M. Lechthaler, G. Gartner), Geowissenschaftlichen Mitteilungen, Technische Universitat Wien, Heft Nr. 52, 33-42.
(32) Frankich, K. (1980): Mathematical Cartography, Part one, Geographic Map Projections. Division of Surveying Engineering, The University of Calgary.
(33) Frischauf, J. (1905): Die Abbildungslehre und deren Anwendung auf Kartographie und Geodäsie. Druck und Verlag von B. G. Teubner, Leipzig.
(34) Gauß, C. F. (1825): Allgemeine Auflösung der Aufgabe: Die Theile einer gegebnen Fläche auf einer andern gegebnen Fläche so abzubilden, dass die Abbildung dem Abgebildeten in den kleinsten Theilen änlich wird. Astronomischen Abhandlungen, herausgegeben von H. C. Schumacher, Heft 3, 1-30, Altona. Also in: Gauß' Werken Bd. IV, 189-216, Göttingen, 1873; Ueber Kartenprojection. Abhandlungen von Lagrange (1779) und Gauß (1822). Herausgegeben von A. Wangerin. Ostwald's Klassiker der exakten Wissenschaften, Nr. 155, 57-101, Verlag von Wilhelm Engelmann, Leipzig.
(35) Gavazzi, A. (1933): Kartografijske projekcije, Akademski geografski klub u Zagrebu, 1-66.
(36) Ginzburg, G. A., Salmanova, T. D. (1957): Atlas dlja vybora kartograficheskih proekcij. Trudy Central'nogo nauchno-issledovatel'skogo instituta geodezii, aeros'emki i kartografii, Vypusk 110, Geodezizdat, Moskva.
(37) Gojčeta, B., Zekušić, S. (1998): Geodetsko-prostorni sustav Republike Hrvatske kao osnova za GIS, GIS Croatia 98, Osijek, Zbornik radova, 19-23.
(38) Graur, A. V. (1956): Matematicheskaja kartografija. Izdatel'stvo Leningradskogo Universiteta, Leningrad.
(39) Hartl, H. (1886): Die Projectionen der wichtigsten vom k. k. Generalquartiermeisterstabe und vom k. k. militärgeographischen Institute

herausgegebenen Kartenwerke, Mittheilungen des k. k. Militär-Geographischen Institutes, VI. Band 1886, Wien 1886, 120-197.
(40) Hartl, H. (1896): Studien über fläschentreue Kegelprojectionen, Mittheilungen des k. und k. Militär-Geographischen Institutes, XV. Band 1895, Wien 1896, 203-249.
(41) Hinks, A. R. (1912): Map Projections. Cambridge University Press, Cambridge.
(42) Hoschek, J. (1969, 1984): Mathematische Grundlagen der Kartographie. Bibliographisches Institut, Mannheim, Zürich.
(43) Hristow, W. K. (1943): Die Gauss-Krüger'schen Koordinaten auf dem Ellipsoid. B. G. Teubner, Leipzig, Berlin.
(44) Hristow, W. K. (1955): Dei Gaußschen und geographischen Koordinaten auf dem Ellipsoid von Krassowsky. VEB Verlag Technik, Berlin.
(45) Hristov, V. K. (1957): Koordinaty Gaussa-Krjugera na ellipsoide vrashchenija. Geodezizdat, Moskva.
(46) Jacobi, C. G. J. (1866): Vorlesungen über Dynamik, nebst fünf hinterlassenen Abhandlungen desselben, herausgegeben von A. Clebsch, Berlin, Druck und Verlag von Georg Reimer, 28. Vorlesung: Die kürzeste Linie auf dem dreiaxigen Ellipsoid. Das Problem der Kartenprojection, 212-220.
(47) Jakić, T. (1916): Crtanje mreža za geografske karte. Nastavni vjesnik, Zagreb, Knjiga XXIV, 353-366, 418-430, 492-503, 544-556.
(48) Kavrayskiy, V. V. (1958): Izbrannye trudy, Tom II, Matematicheskaja kartografija, Vypusk 1, Obshchaja teorija kartograficheskih proekcij. Izdanie Upravlenija nachal'nika Gidrograficheskoj sluzhby VMF.
(49) Kavrayskiy, V. V. (1959): Izbrannye trudy, Tom II, Matematicheskaja kartografija, Vypusk 2, Konicheskie i cilindricheskie proekcii, ih primenenie. Izdanie Upravlenija nachal'nika Gidrograficheskoj sluzhby VMF.
(50) Kavrayskiy, V. V. (1960): Izbrannye trudy, Tom II, Matematicheskaja kartografija, Vypusk 3, Perspektivnye, krugovye i drugie vazhnejshie proekcii. Navigacionnye zadachi. Izdanie Upravlenija nachal'nika Gidrograficheskoj sluzhby VMF.
(51) Kozličić, M. (1995): Kartografski spomenici hrvatskoga Jadrana. Izbor karata, planova i veduta do kraja 17. stoljeća. AGM, Zagreb.
(52) Kuntz, E. (1983): Kartennetzentwurfslehre. Herbert Wichmann Verlag, Karlsruhe.
(53) Kurepa, S., Kraljević, H. (1986): Matematička analiza, 4. dio, Funkcije kompleksne varijable. Tehnička knjiga, Zagreb.
(54) Lago, L., Rossit, C. (1981): Descriptio Histriae. Trieste.
(55) Lago, L. (1996): Stare karte Jadrana, C.A.S.H., Pula.
(56) Lagrange, J. L. de (1781): Sur la construction des cartes géographiques, Nouveaux Mémoires de l'Académie Royale des Sciences et Belles Lettres de Berlin, Année 1779, 161-210 (Berlin 1781). Also in: Oeuvres de Lagrange, T. IV, 635-692, and in: Ueber Kartenprojection. Abhandlungen von Lagrange (1779) und Gauß (1822). Herausgegeben von A. Wangerin. Ostwald's Klassiker der exakten Wissenschaften, Nr. 155, Verlag von Wilhelm Engelmann, Leipzig, 3-56.
(57) Lambert, J. H. (1772): Beiträge zum Gebrauche der Mathematik und deren Anwendungen, Dritter Theil, im Verlag der Buchhandlung der Realschule, Berlin. Translated in English with an introduction by W. R. Tobler: Notes and Comments on the Composition of Terrestrial and Celestial Maps, Michigan Geographical Publication No. 8, Department of Geography, University of Michigan, Ann Arbor, 1972.
(58) Lapaine, M., Frančula, N. (1992): Modificirana Gilbertova projekcija, Kompjutor u obnovi Hrvatske, CAD FORUM'92, CAD Sekcija Saveza društava arhitekata Hrvatske, Zagreb, Zbornik radova, 159-164.

(59) Lapaine, M., Frančula, N. (1993): Gilbert Two-World Projection, Proceedings of the 16th International Cartographic Conference, Cologne/Köln, Vol. 1, Bielefeld, 66-82.
(60) Lapaine, M., Frančula N. (1993): Usporedba dviju Gauß-Krügerovih projekcija za područje Hrvatske, manuscript.
(61) Lapaine, M., Sudeta, N., Frančula, N. (1994): Gilbert's Globe, 6th International Conference on Engineering Computer Graphics and Descriptive Geometry, Otsuma Women's University, Tokyo, Proceedings, Vol. 1, 154-158.
(62) Lapaine, M., Sudeta, N., Frančula, N., Vdović, R. (1995): Gilbert Projection and Gilbert Globe, in: Kremers, H., Pillmann, W. (eds.): Raum und Zeit in Umweltinformationssystemen, Space and Time in Environmental Information Systems, Metroplois-Verlag, Marburg, Teil II/Part II, 870-871.
(63) Lapaine, M., Tutić, D. (1999): Hrvatska državna kartografska projekcija, simpozij Državne geodetske osnove i zemljišni informacijski sustavi, Opatija, Hrvatsko geodetsko društvo, Zbornik radova, 155-164.
(64) Lapaine, M. (2000): Proposal for Official Map Projections of the Republic of Croatia, State Geodetic Administration of the Republic of Croatia, University of Zagreb, Faculty of Geodesy, Zagreb.
(65) Li Changming (1979): On the classification of map projections, Acta geographica Sinica, 34, 139-155.
(66) Lee, L. P. (1976): Conformal Projections Based on Elliptic Functions. Cartographica Monograph 16, Supp. 1 to Canadian Cartographer 13.
(67) Lovrić, P. (1988): Opća kartografija. Sveučilišna naklada Liber, Zagreb.
(68) Maling, D. H. (1973): Coordinate Systems and Map Projections. George Philip and Son Ltd., 2nd ed., Pergamon Press, Oxford.
(69) Marek, J. (1875): Technische Anleitung zur Ausführung der trigonometrischen Operationen des Katasters. A Magyar Királyi Államanyomdából, Budapest.
(70) Marković, M. (1993): Descriptio Croatiae. Naprijed, Zagreb.
(71) Marković, Ž. (1965): Uvod u višu analizu, II. dio, 1. i 2. svezak. Sveučilište u Zagrebu.
(72) Maurer, H. (1935): Ebene Kugelbilder. Ein Linnésches System der Kartenentwürfe. Petermanns Mitteilungen, Ergänzungsheft 221.
(73) Melluish, R. K. (1931): An Introduction to the Mathematics of Map Projections. Cambridge University Press, Cambridge.
(74) Merkel, H. (1956, 1958): Grundzüge der Kartenprojektionslehre, Teil I, Die theoretischen Grundlagen, Teil II, Abbildungsverfahren. Deutsche Geodätische Kommission, Reihe A, Heft 17 und 17/II.
(75) Meshcherjakov, G. A. (1968): Teoreticheskie osnovy matematicheskoj kartografii. Izdatel'stvo Nedra, Moskva.
(76) Moellering, H. (1980): Strategies of Real-Time Cartography, Cartographic Journal, 17 (1), 12-15.
(77) Moellering, H. (ed, 1991): Spatial Database Transfer Standards: Current International Status, ICA, Elsevier Applied Science, London, New York.
(78) Moellering, H. (1991): Whither Analytical Cartography, Cartography and Geographic Information Systems, Vol.18, No.1, 7-9.
(79) Morrison, J. L. (1974): Changing Philosophical-Technical Aspects of Thematic Cartography, American Cartographer, 1 (1), 5-14.
(80) Murdoch, P. (1758): On the best form of geographical maps. Philosophical Transactions of the Royal Society, Vo. 50, No. 2, 553-562.
(81) Narobe, Z. (1975): Pojednostavljen način računanja konvergencije meridijana za potrebe žiroskopske orijentacije, Geodetski list, 68-73.
(82) Niče, V. (1971): Perspektiva. Školska knjiga, Zagreb.

(83) Paj, R. (ed, 1995): Opći digitalni zemljovid Republike Hrvatske mjerila 1:300 000, prvo privremeno izdanje, Radna grupa Zavoda za fotogrametriju d.d., Zagreb.
(84) Pandžić, A. (1987): Stare karte i atlasi Povijesnog muzeja Hrvatske. Katalog muzejskih zbirki XXII, Zagreb.
(85) Pandžić, A. (1988): Pet stoljeća zemljopisnih karata Hrvatske. Katalog izložbe Povijesnog muzeja Hrvatske, Zagreb.
(86) Pandžić. A. (1992): Hrvatska i njene granice na starim kartama. In: Granice Hrvatske na zemljovidima od 12. do 20. stoljeća, katalog izložbe, Zagreb.
(87) Pavlov, A. A. (1972): Nekatorye voprosy matematicheskoj kartografii. Geodezija i kartografija 2, 65-68.
(88) Pavlov, A. A. (1974): Prakticheskoe posobie po matematicheskoj kartografii. Izdatel'stvo Leningradskogo universiteta, Leningrad.
(89) Rajtr, J. (1914): Općenite zasade kartografije. Vijesnik, glasilo Udruženja civilnih tehnika kraljevina Hrvatske i Slavonije, br. 8, 118-121.
(90) Randić, L. (1953): O konstrukciji vertikala u stereografskoj projekciji, Geodetski list, 299-306.
(91) Reignier, F. (1957): Les systèmes de projection et leurs applications, Tome I, Texte, Tome II, Graphiques. Publications techniques de l'Institute Géographique National, Paris.
(92) Richardus, P., Adler, R. K. (1972): Map Projections for Geodesists, Cartographers and Geographers. North Holland Publishing, Amsterdam.
(93) Savezna geodetska uprava (1953): Osnovni geodetski radovi u F. N. R. Jugoslaviji. Beograd.
(94) Segen, D. (1881): O crtanju mreža za geografijske karte. Izviešće o Kr. velikoj realci u Osieku koncem školske godine 1880/1, 3-44.
(95) Segen, D. (1893): Osnove relijef-perspektive. Nastavni vjesnik, Knjiga I., 209-227.
(96) Sijmons, K. (1992): Cartographic representation and production of GIS data, Proceedings of the First International Conference on Surveying and Mapping, Tehran, Vol.2., 117-125.
(97) Sirks, A. (1936): Triangulacije u geodetsko-geometarskoj praksi u raznim projekcionim i koordinatnim sistemima na teritoriji Kraljevine Jugoslavije. Geometarski i geodetski glasnik 2, 101-120.
(98) Snyder, J. P. (1982): Map Projections Used by U. S. Geological Survey. U. S. Geological Survey Bulletin 1532, 2nd ed., Washington.
(99) Snyder, J. P. (1985): Computer-Assisted Map Projection Research. U. S. Geological Survey Bulletin 1629, Washington.
(100) Snyder, J. P. (1987): Map Projections: A Working Manual. U. S. Geological Survey Professional Paper 1395, Washington.
(101) Snyder, J. P. (1993): Flattening the Earth, Two Thousand Years of Map Projections. The University of Chicago Press, Chicago, London.
(102) Snyder, J. P., Steward, H. (eds, 1988): Bibliography of Map Projections. U. S. Geological Survey Bulletin 1856, Washington.
(103) Solarić, R. (1980): Računanje koordinata točaka u Mercatorovoj projekciji primjenom džepnog računala, Geodetski list, 181-187.
(104) Solov'ev, M. D. (1946): Kartograficheskie proekcii. Izdatel'stvo geodezicheskoj i kartograficheskoj literatury, GUGK pri SNK SSSR, Moskva.
(105) Solov'ev, M. D. (1969): Matematicheskaja kartografija. Izdatel'stvo Nedra, Moskva.
(106) Sowton, M. (1991): Development of GIS-related activities at the Ordnance Survey. In: J. Maguire, M. F. Goodchild, D. W. Rhind (eds.): Geographical Information Systems, Vol. 2 - Application, Longman, London, 23-38.

(107) Steers, J. A. (1970): An Introduction to the Study of Map Projections. 15th edition, University of London Press, London.
(108) Steinhauser, A. (1857): Grundzüge der mathematischen Geographie und der Landkartenprojection. Friedrich Beck's Universitäts-Buchhandlung, Wien.
(109) Teng, A. T. (1991): Building geographic data bases with input from federal digital data files, Technical Papers of 1991 ACSM-ASPRS annual convention, Baltimore 1991, Vol.4 GIS, 208-216.
(110) Thomas, P. D. (1952): Conformal Projections in Geodesy and Cartography. U. S. Coast and Geodetic Survey Special Publication 251, Washington.
(111) Tissot, A. (1881) Mémoire sur la représentation des surfaces et les projections des cartes géographiques. Gauthier-Villars, Paris.
(112) Tissot, A. (1887) Die Netzentwürfe geographischer Karten (Autorisierte Deutsche bearbeitung mit einigen Zusätze besorgt von E. Hammer). J. B. Metzlersche Buchhandlung, Stuttgart.
(113) Tobler, W. R. (1965): Medieval distortions: the projections of ancient maps, Annals, Association of American Geographers, 56 (2) 351-360.
(114) Tobler, W. R. (1976): The geometry of mental maps, in: R. Golledge and G. Rushton, eds., Essays on the Multidimensional Analysis od Perceptions and Preferences, Columbus, Ohio State University Press, 69-81.
(115) Tober, W. R. (1994): Bidimensional regression, Geographical Analysis, 26, 117-120.
(116) Urmajev (1941): Matematicheskaja kartografija, Izdatel'stvo Vojennoinzhenernoj Akademii, Moskva.
(117) Vahramejeva, L. A., Bugajevskij, L. M., Kazakova, Z. L. (1986): Matematicheskaja kartografija. Nedra, Moskva.
(118) Villi, M. (1993): Informacijski sustav okoliša Republike Hrvatske, Pilot projekt "Demografija", CAD Forum '93, Zagreb, Zbornik radova, 5-10.
(119) Vranić, V. (1927): O izvođenju formula sferne trigonometrije s pomoću stereografske projekcije. Nastavni vjesnik, knjiga XXXVI, sv. 1-2. Also: Über die Ableitung der Formeln der sphärischen Trigonometrie mit Hilfe der stereographischen Projektion.
(120) Wagner, K.-H. (1949): Kartographische Netzentwürfe. Bibliographisches Institut, Leipzig.
(121) Wang Qiao et al. (1996): Methods for Variable Scale Map Projections, Publishing House of WTUSM.
(122) Yang Qihe (1987): The perspective azimuthal projection under variable view points, Acta Geodetica et Cartographica Sinica, 16, 306-131.
(123) Yang, Q., Snyder, J. P., Tobler, W. R. (2000): Map Projection Transformation, Priciples and Applications, Taylor & Francis, London.
(124) Yang Qihe, Yang Xiaomei (1997): Three types of latitudinal function and linear interpolation algorithms used in surveying and cartography, Acta Geodetica et Cartographica Sinica, 26, 92-95.
(125) Young, A. E. (1920): Some Investigations in the Theory of Map Projections. Royal Geographical Society, Technical Series 1, London.