

# Standard, Equidistant and Secant Parallels

or lack of critical thinking  
or about an illusion in map projections

Miljenko Lapaine

The aim of this presentation:

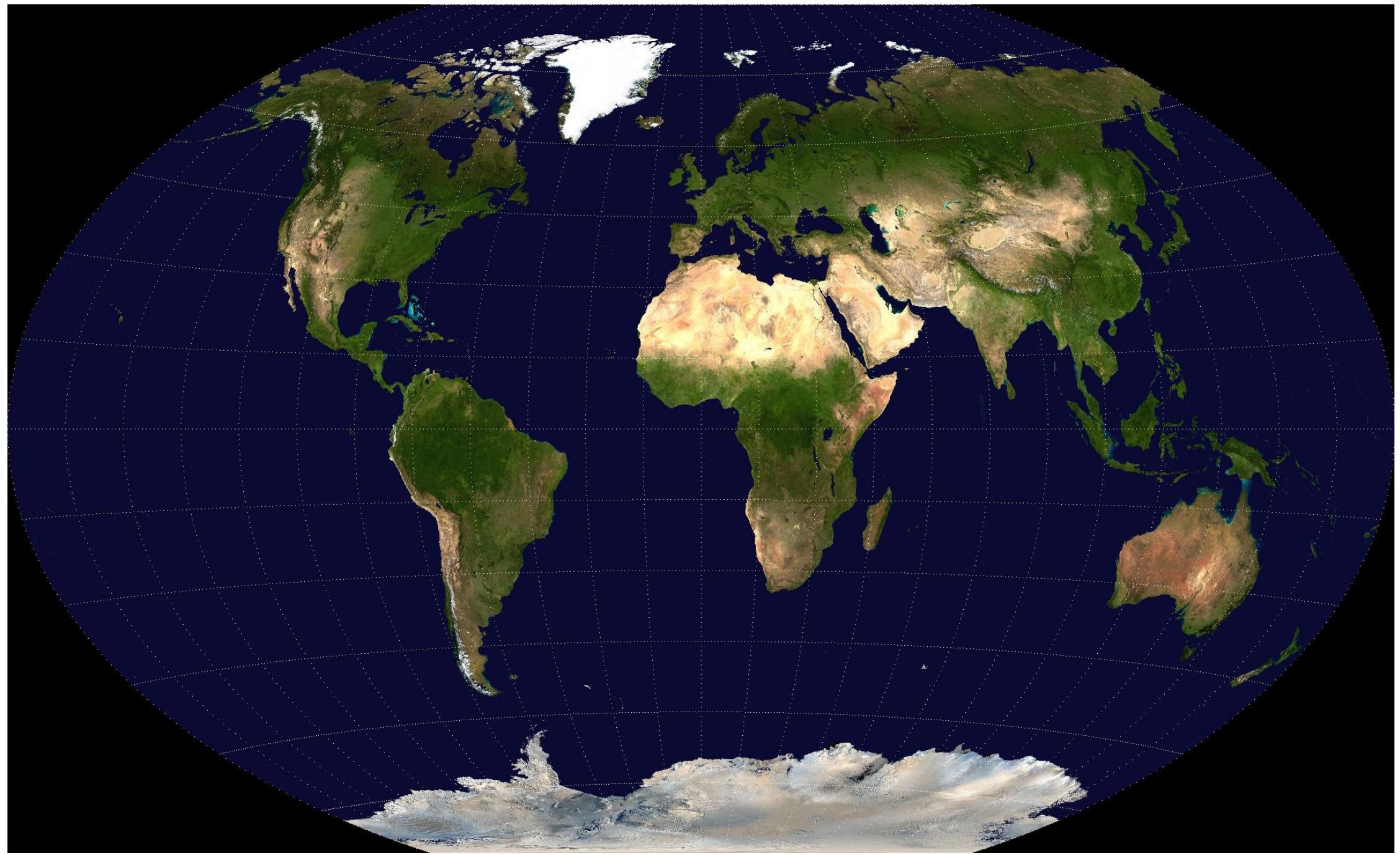
To show some problems we encountered and

The need to continue the fight against illusions in the theory of map projections



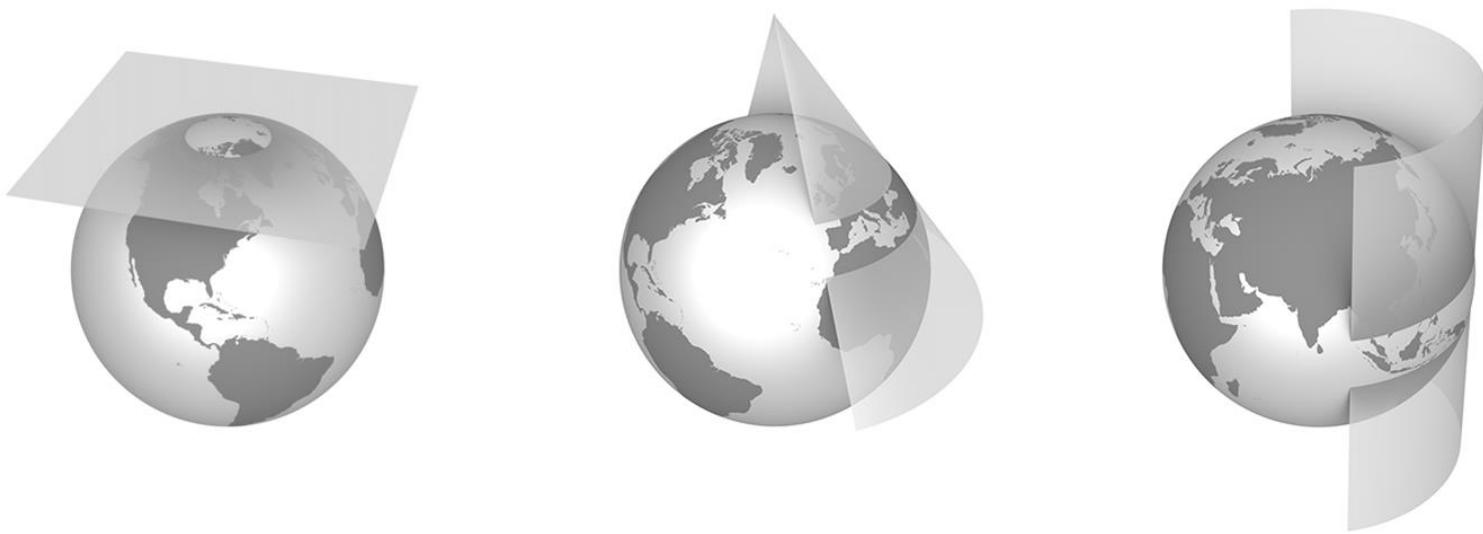
*Globus je  
najvjerniji  
prikaz  
Zemlje*

Globe is the most  
faithful representation  
of the Earth



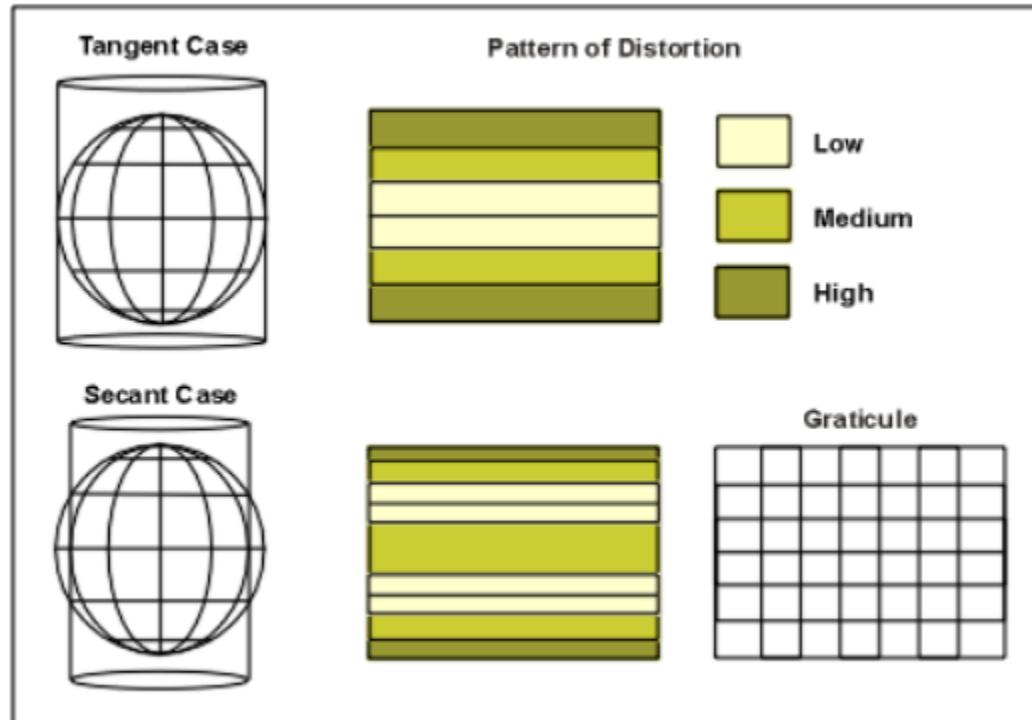
Map projections and maps are ubiquitous

- atlases
- textbooks
- encyclopedias
- Internet
- ...



Pandemic in cartography

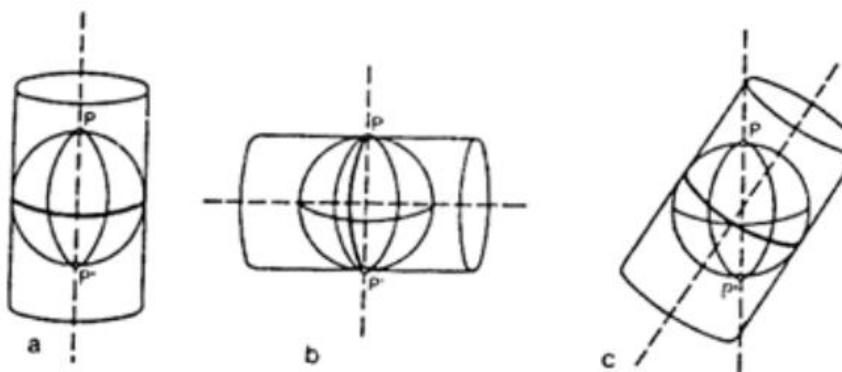
# Cylindrical projections



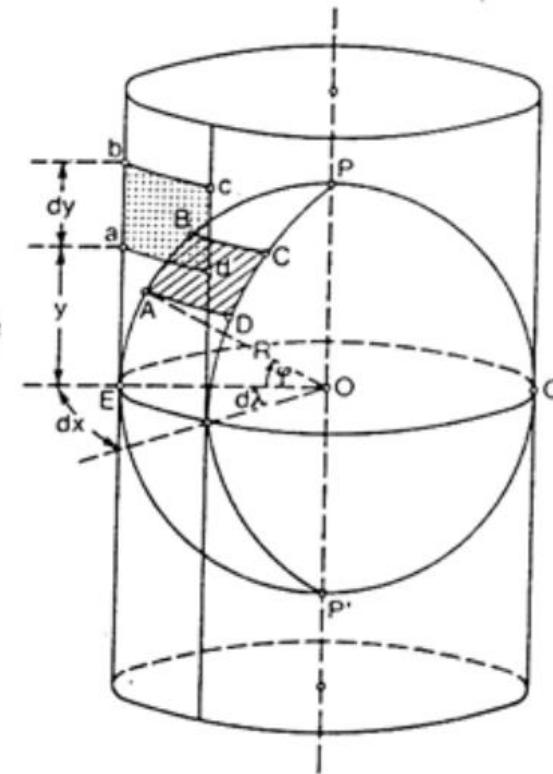
Izvornik: [www.fes.uwaterloo.ca/crs/geog165/cylproj.htm](http://www.fes.uwaterloo.ca/crs/geog165/cylproj.htm)

[http://www2.geof.unizg.hr/~nvucetic/OGI\\_kart\\_proj.pdf](http://www2.geof.unizg.hr/~nvucetic/OGI_kart_proj.pdf)

# Cylindrical projections

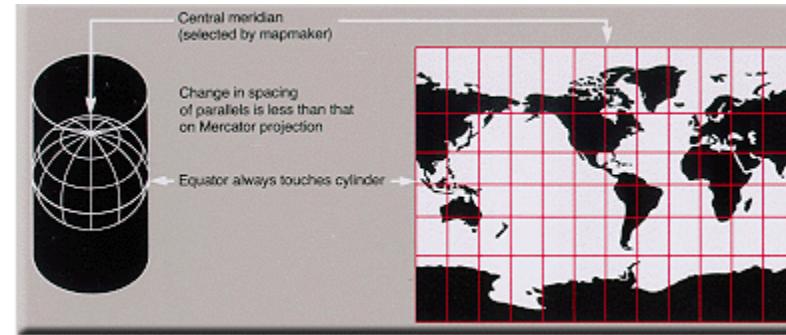


CILINDRIČNE PROJEKCIJE  
a = prava (uspravna); b = poprečna (ekvatorska); c = kosa



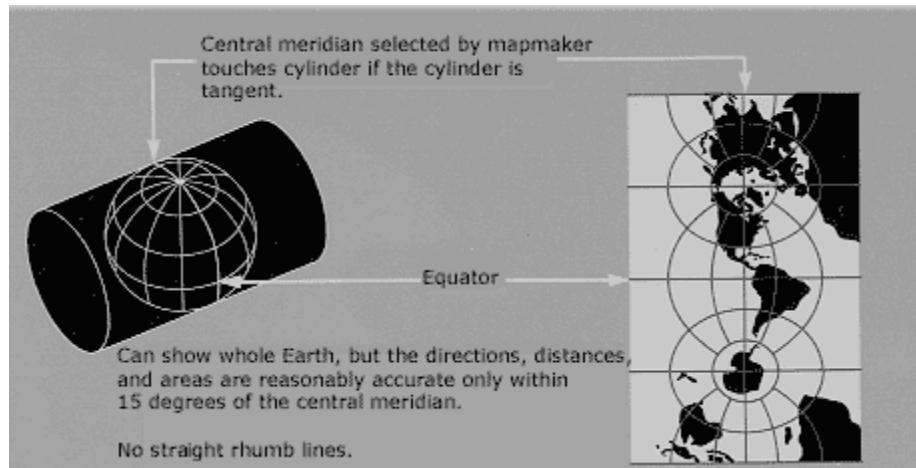
<http://www.pfri.uniri.hr/~brsic/downloads/3.%20TERESTRICKA%20NAVIGACIJA%20Kartografske%20projekcije.pdf>

# Cylindrical projections



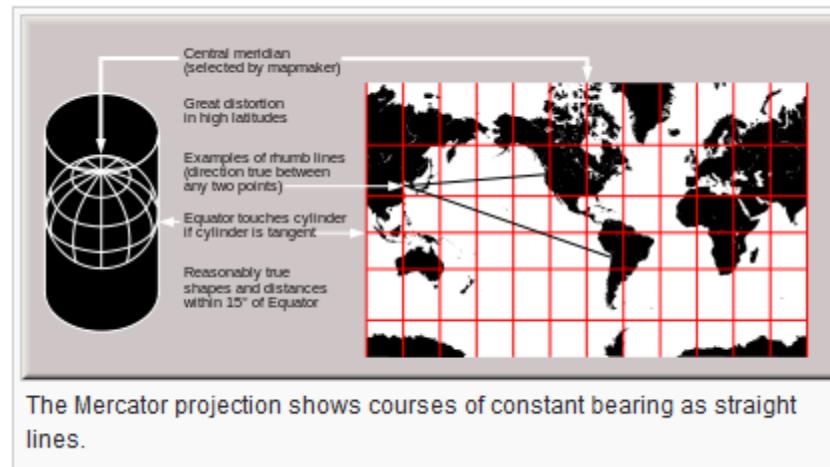
[https://hr.wikipedia.org/wiki/Mercatorova\\_projekcija](https://hr.wikipedia.org/wiki/Mercatorova_projekcija)

# Cylindrical projections



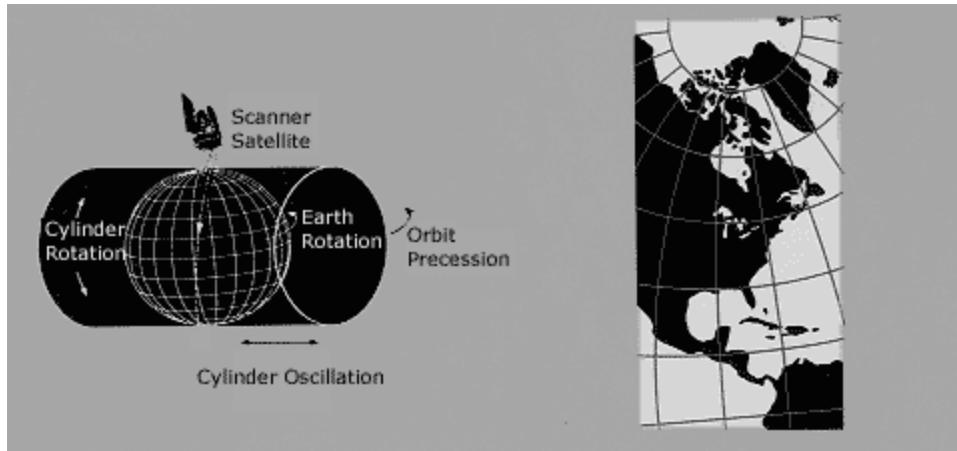
[https://en.wikipedia.org/wiki/Map\\_projection](https://en.wikipedia.org/wiki/Map_projection)

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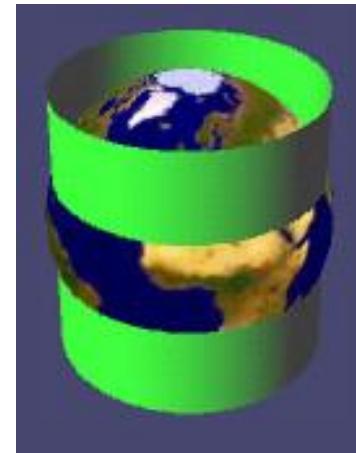
<http://egsc.usgs.gov/isb//pubs/MapProjections/projections.html>

# Cylindrical projections



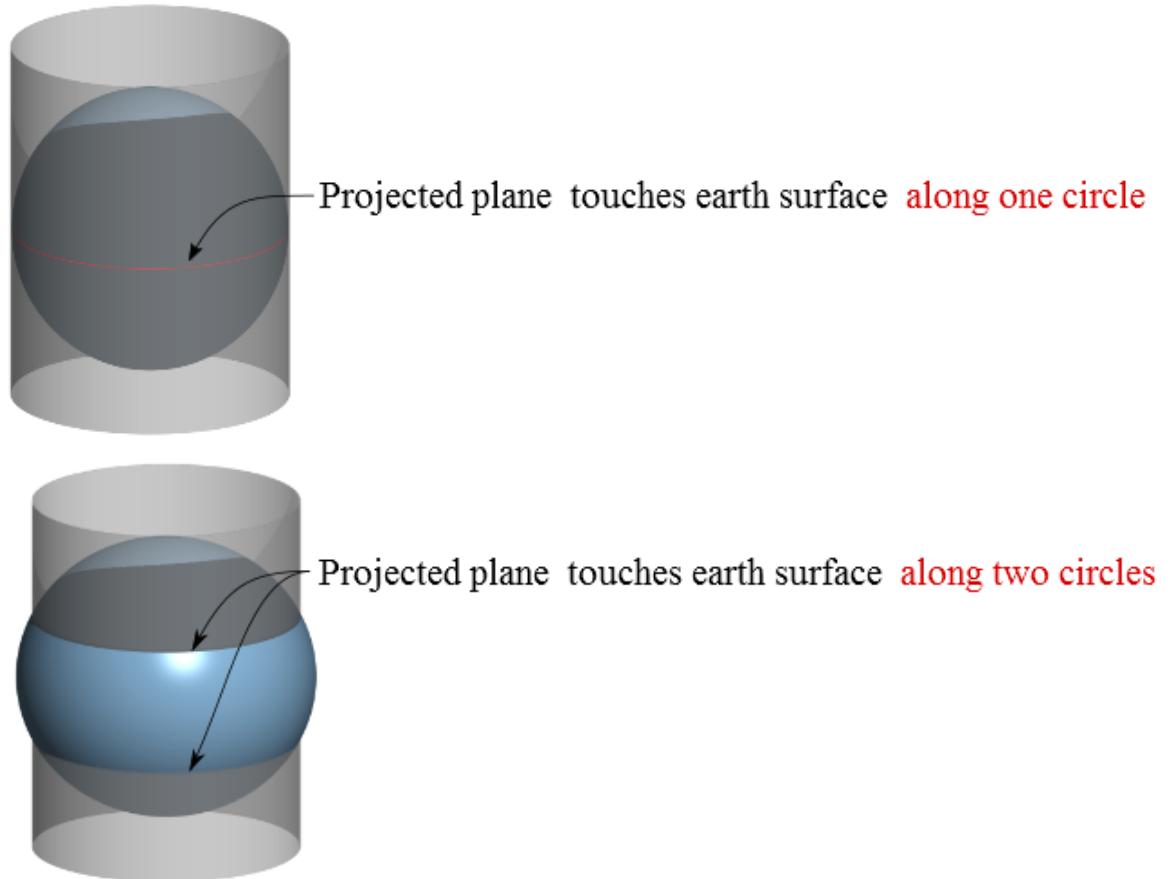
<http://www.slideshare.net/yourmohsin/projections-and-coordinate-system>

# Cylindrical projections



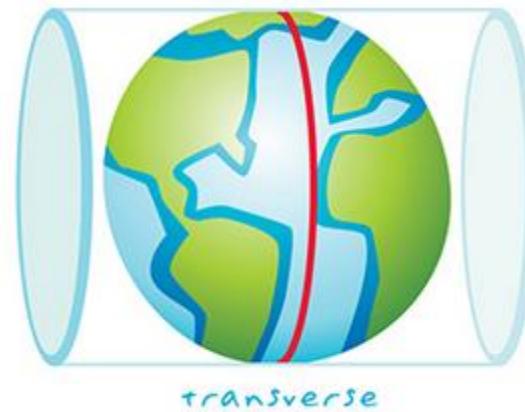
<http://lazarus.elte.hu/~guszlev/vet/cylin.htm>

# Cylindrical projections



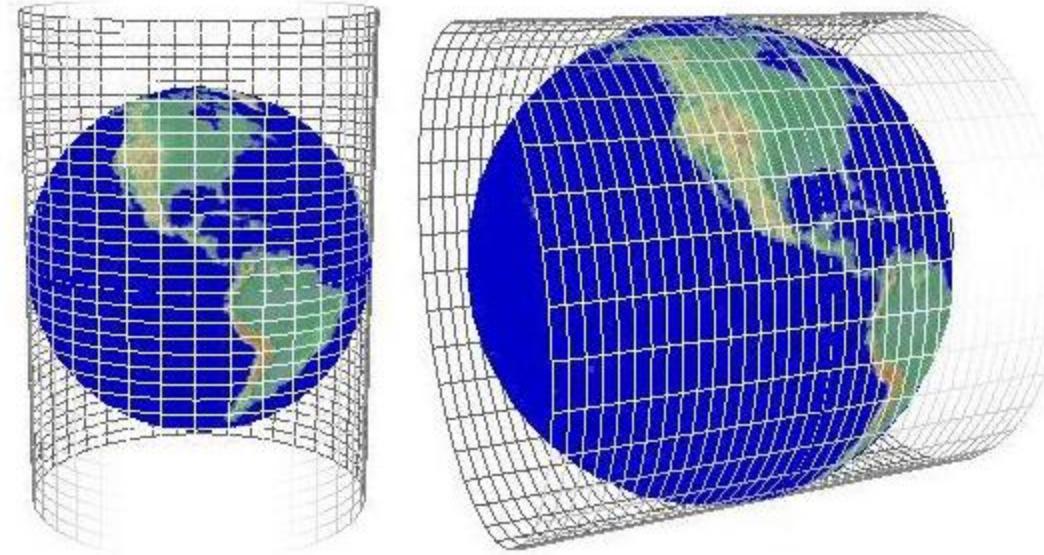
<https://mgimond.github.io/Spatial/coordinate-systems.html>

# Cylindrical projections



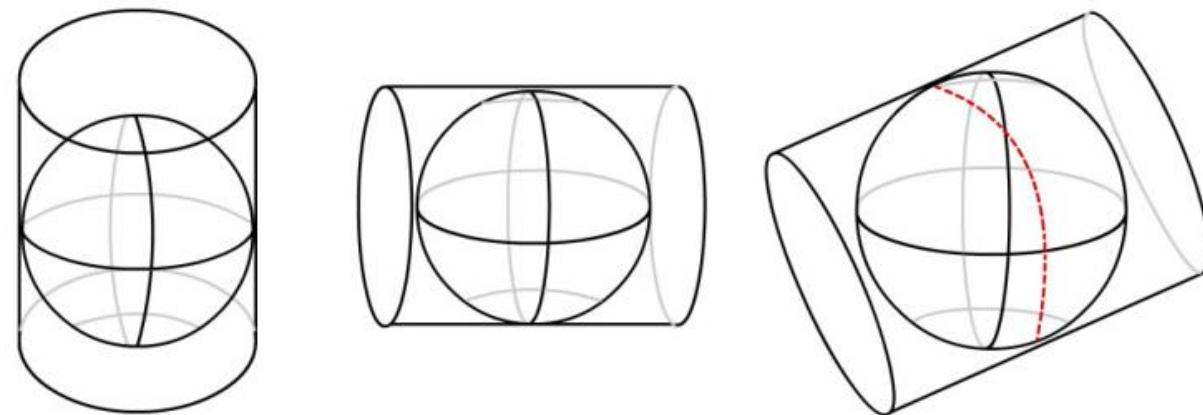
<http://support.esri.com/other-resources/gis-dictionary/term/cylindrical%20projection>

# Cylindrical projections



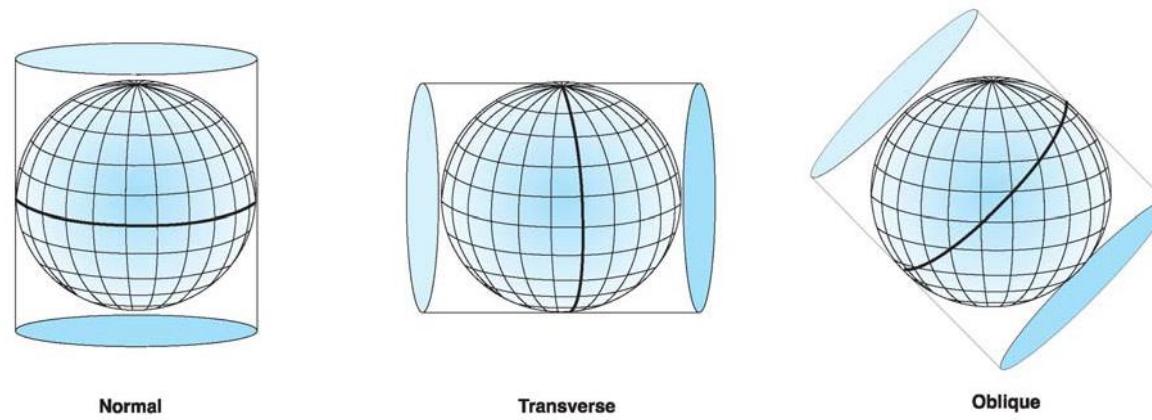
<http://www.codeguru.com/cpp/g-m/bitmap/viewers/article.php/c9187/2D--3D-Visualization-Techniques-for-GeoReferenced-Images.htm>

# Cylindrical projections



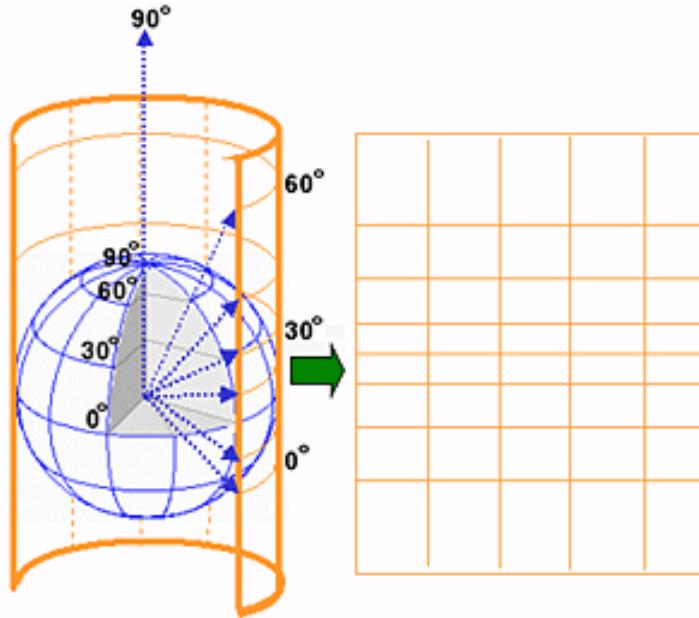
<http://nptel.ac.in/courses/105102015/42>

# Cylindrical projections



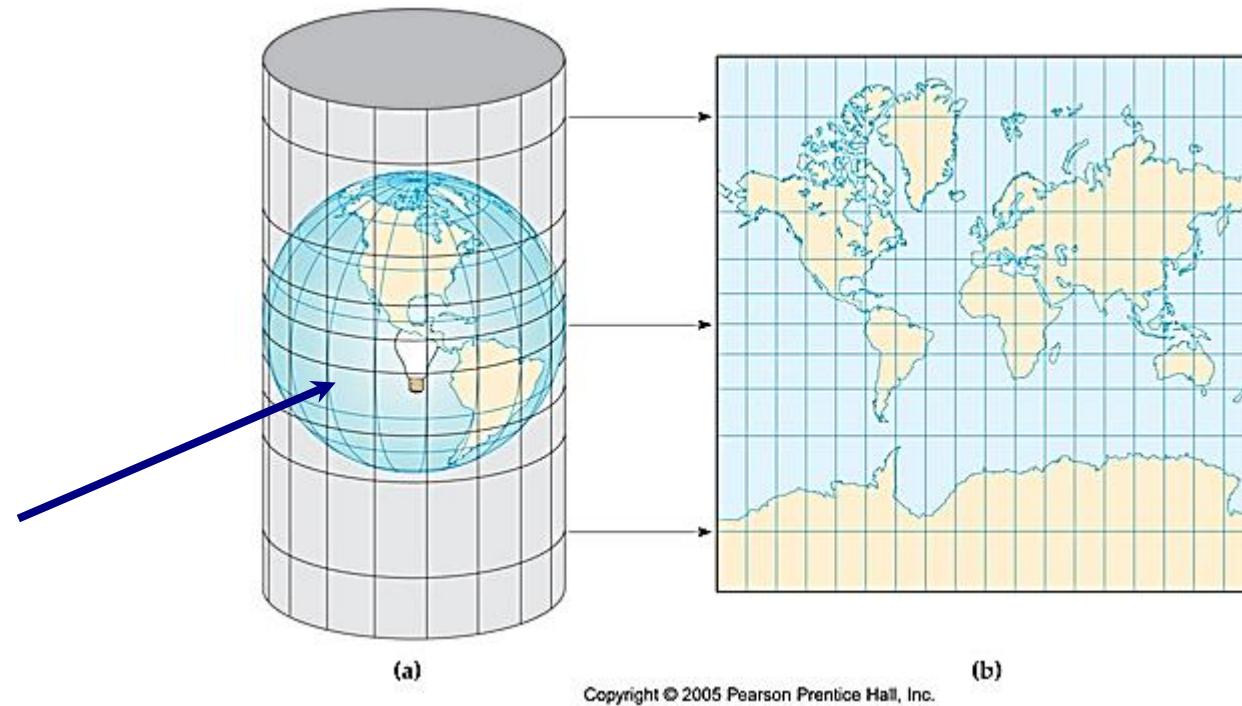
<http://maps.unomaha.edu/Peterson/gis/notes/MapProjCoord.html>

# Cylindrical projections

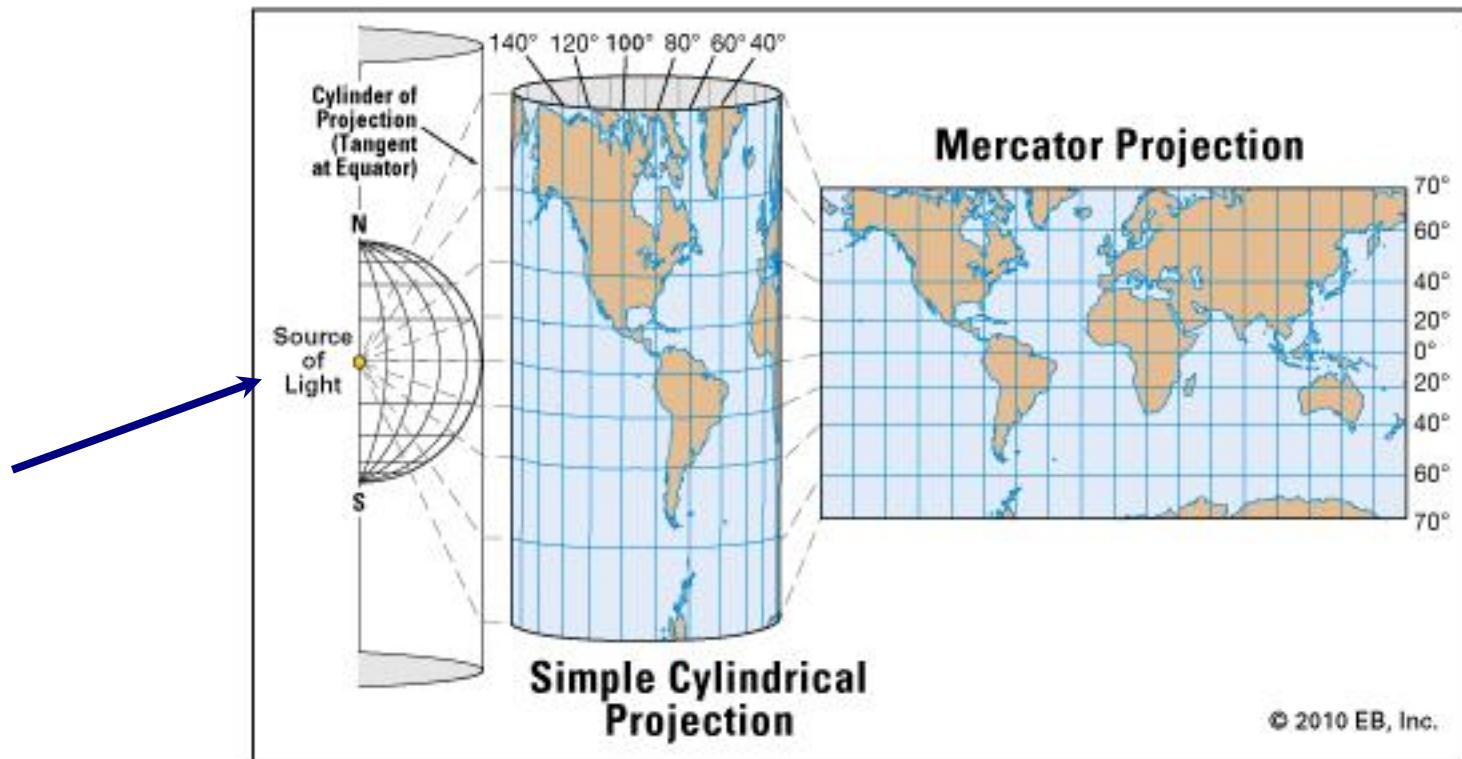


<http://www.geography.hunter.cuny.edu/~jochen/GTECH361/lectures/lecture04/concepts/Map%20coordinate%20systems/Cylindrical%20projections.htm>

# Cylindrical projections



[http://web.gccaz.edu/~lnewman/gph111/topic\\_units/systems\\_grid\\_proj/systems\\_time/systems\\_time2.html](http://web.gccaz.edu/~lnewman/gph111/topic_units/systems_grid_proj/systems_time/systems_time2.html)



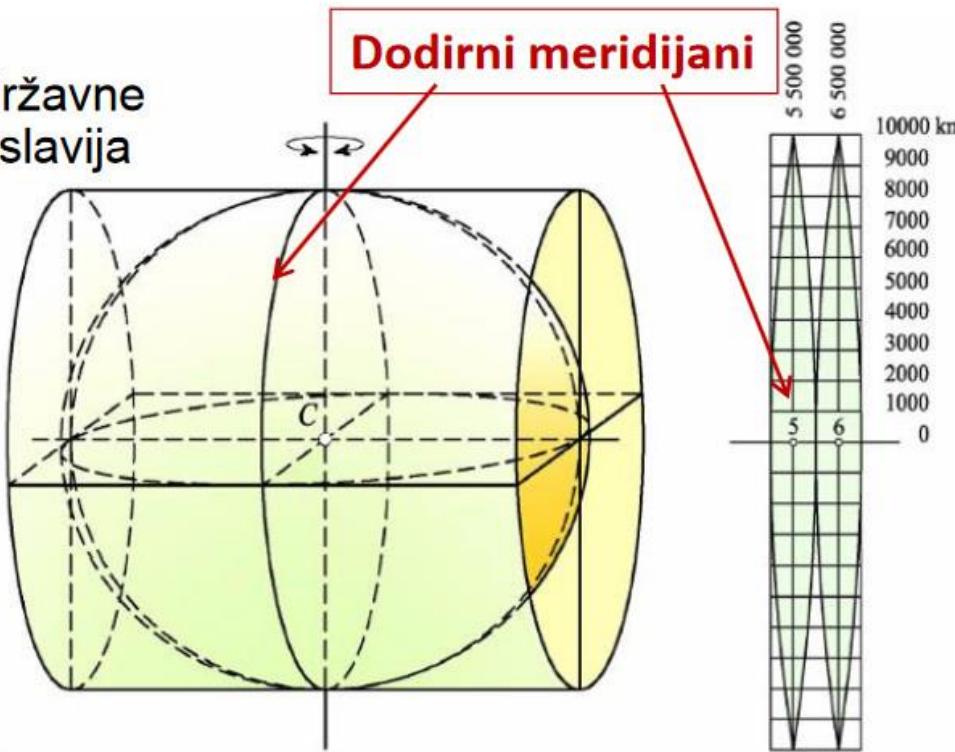
The Mercator projection—a transformation from the simple cylindrical projection—is used for navigation, since lines of constant direction on the Earth appear as straight lines on the map.

- **Gauss-Krüger projekcija**

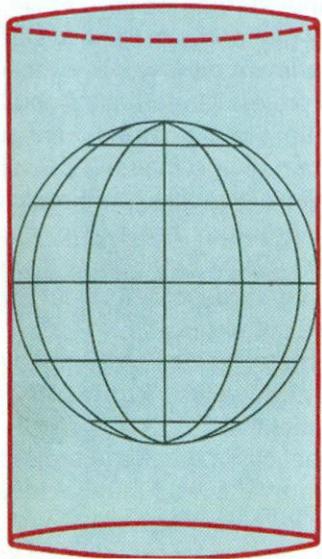
- Austrija je još 1917. uvela za potrebe državne izmjere, kasnije Njemačka i bivša Jugoslavija

- Konformna poprečna cilindrična projekcija
- Elipsoid – Bessel 1841

- Dijeli se na zone široke  $3^{\circ}$  unutar kojih su deformacije male
- Svaka zona ima drugi dodirni meridian
  - $15^{\circ} / 3^{\circ} = 5^{\circ} \Rightarrow$  zona 5
  - $18^{\circ} / 3^{\circ} = 6^{\circ} \Rightarrow$  zona 6
- U svakoj zoni je projekcija središnjeg meridijana je **os x koordinatnog sustava, a os y je projekcija ekvatora**
- y koordinata se uvećava za **500 000 m** zbog izbjegavanja negativnih vrijednosti



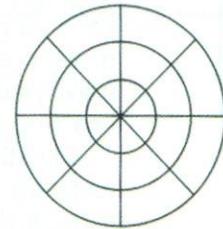
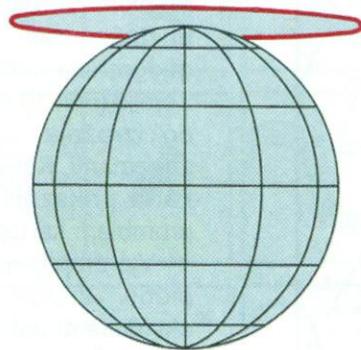
### **VALJKASTE**



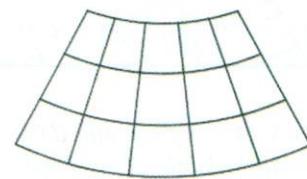
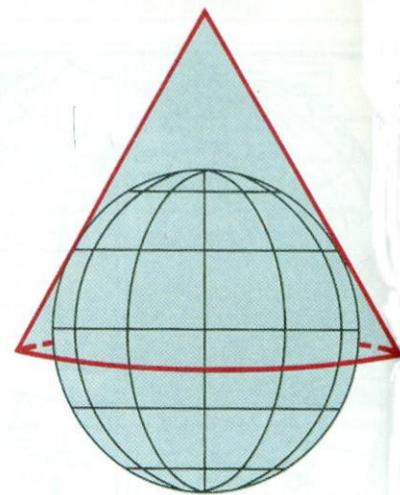
Mreža

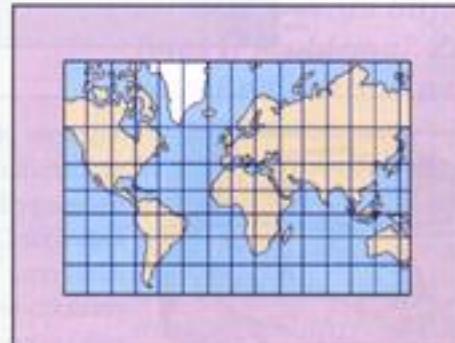


### **HORIZONTSKE**



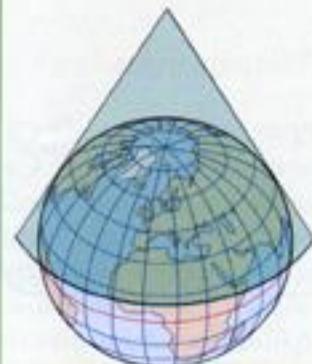
### **STOŽASTE**





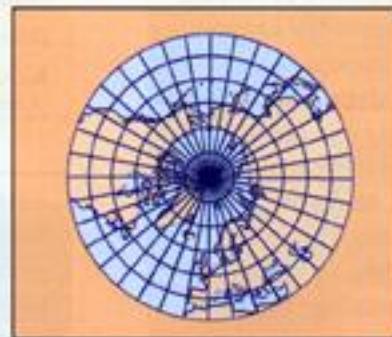
### Valjkasta projekcija

Valjak dodiruje globus na ekvatoru. Tamo su površine najvjernije, a na polovima su najizobličenije.



### Stožasta projekcija

Stožac dodiruje globus na jednoj paraleli. Tu su površine najtočnije prikazane. Dijelovi stošca koji su više odvojeni od globusa, više su i izobličeni.



### Horizontalna projekcija

Ravna ploha dodiruje globus u jednoj točki. Što su površine udaljenije od te središnje točke, to su izobličenije.

## Map Projections

### Map Projection Classes: Introduction

Introduction

Earth's  
Graticule

Map Projection  
Properties

Map Projection  
Classes

Using Map  
Projections

Introduction

Cylindrical Projections

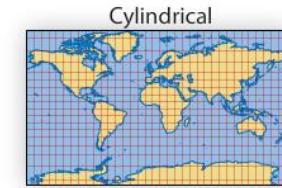
Planar  
Projections

Conic  
Projections

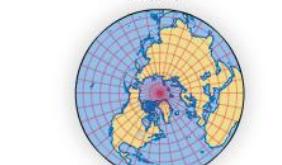
Pseudocylindrical  
(Oval) Projections

SHOW TEXT 

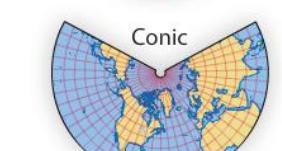
Cylinder



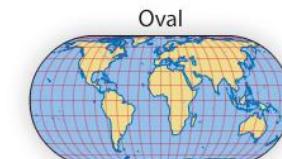
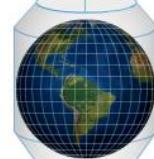
Plane



Cone



Oval



00:23

00:23

REPLAY

PREVIOUS

PAUSE

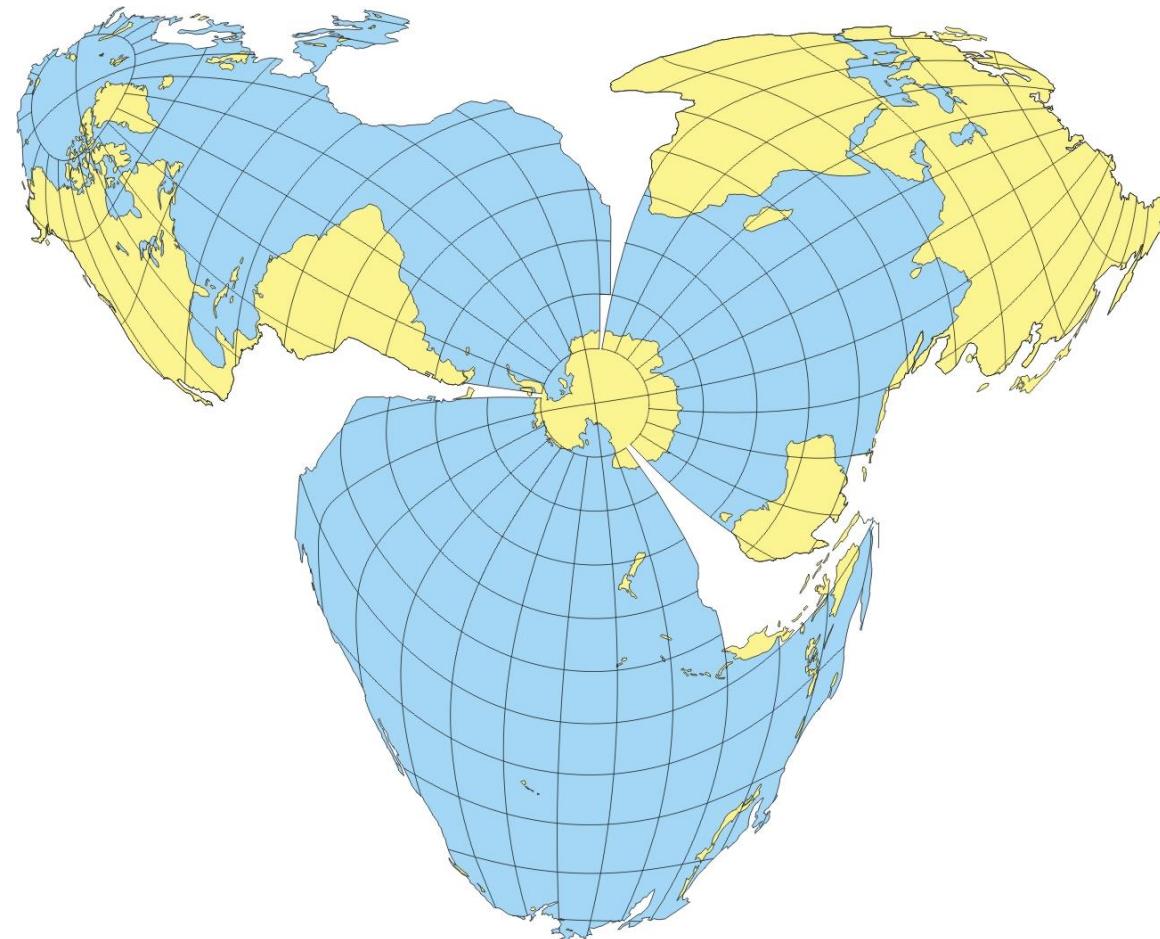
NEXT



Prof. Dr. at the Faculty of Electrical Engineering and Computing:

In addition to the cylindrical surface, cone, pseudo and sinusoidal cylindrical surfaces and planes can be used for projection.





Pseudosomething ...

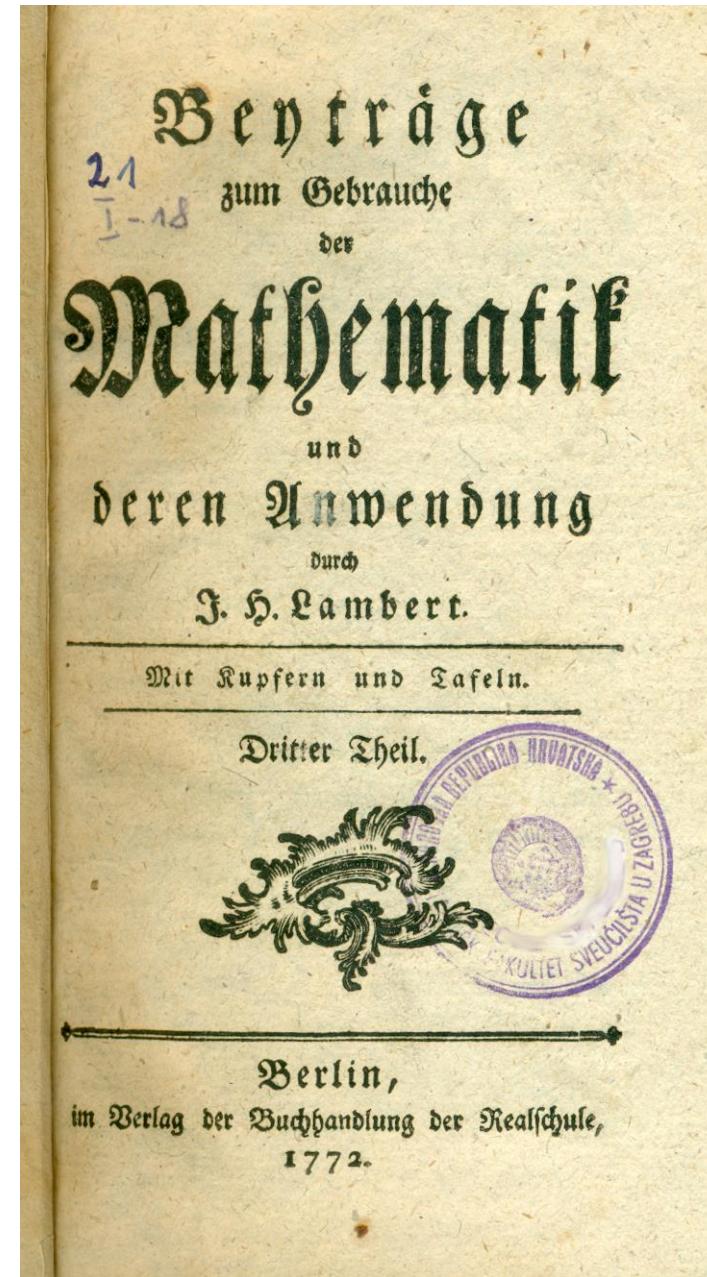
It is obviously a pandemic

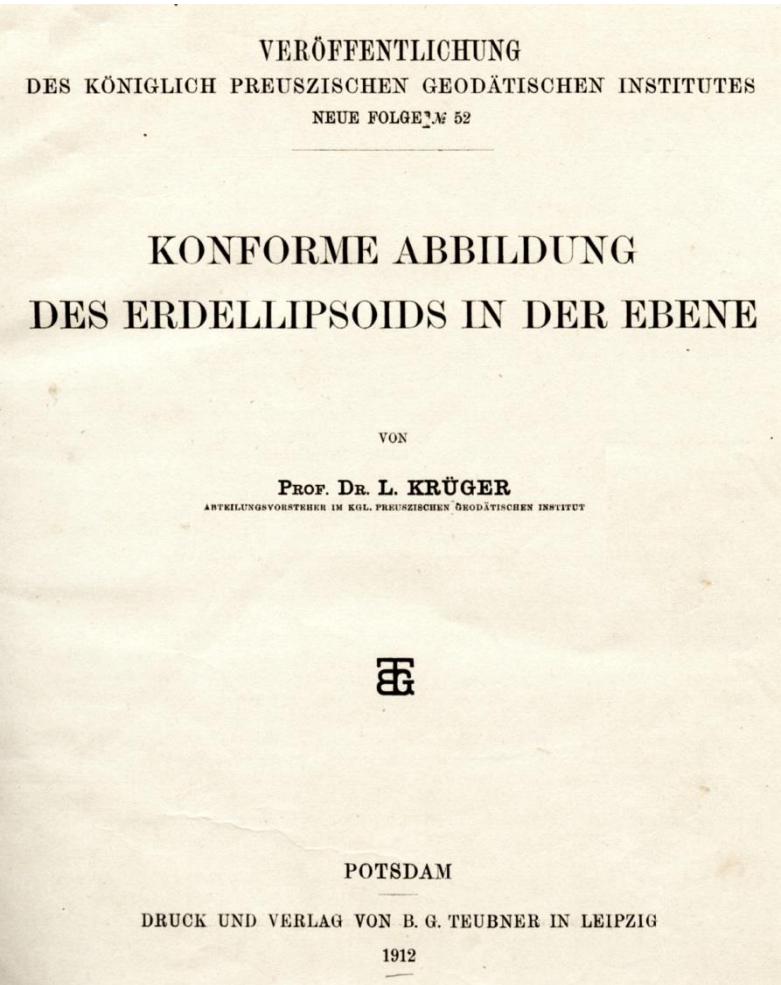
For information on indications, precautions and side effects  
ask your doctor or pharmacist





Contributions to the Use of Mathematics  
and its Applications





Close, C. F. and Clarke, A. R., (1911): Map projections:  
*Encyclopaedia Britannica*, 11th ed., v. 17, p. 653-663,  
reprintings to 1960

Conical projections are those in which the parallels are represented by concentric circles and the meridians by equally spaced radii.  
There is no necessary connexion between a conical projection and any touching or secant cone.

The name conical is given to the group embraced by the above definition, because, as is obvious, a projection so drawn can be round to form a cone.

Lee, L. P. (1944): The Nomenclature and Classification of Map Projections, Empire Survey Review, No. 51, Vol. VII, 190–200.

*Cylindric*: projections in which the meridians are represented as a system of equidistant parallel straight lines, and the parallels by a system of parallel straight lines at right angles to the meridians.

*Conic*: projections in which the meridians are represented as ...

*Azimuthal*: projections in which the meridians are represented as ...

No cylinders, no cones, ... ?!

"No reference has been made in the above definitions to cylinders, cones or planes. The projections are termed cylindric or conic because they can be regarded as developed on a cylinder or cone, as the case may be, but it is as well to dispense with picturing cylinders and cones, since they have given rise to much misunderstanding.

Particularly is this so with regards to the conic projections with two standard parallels: they may be regarded as developed on cones, but they are cones which bear no simple relationship to the sphere."

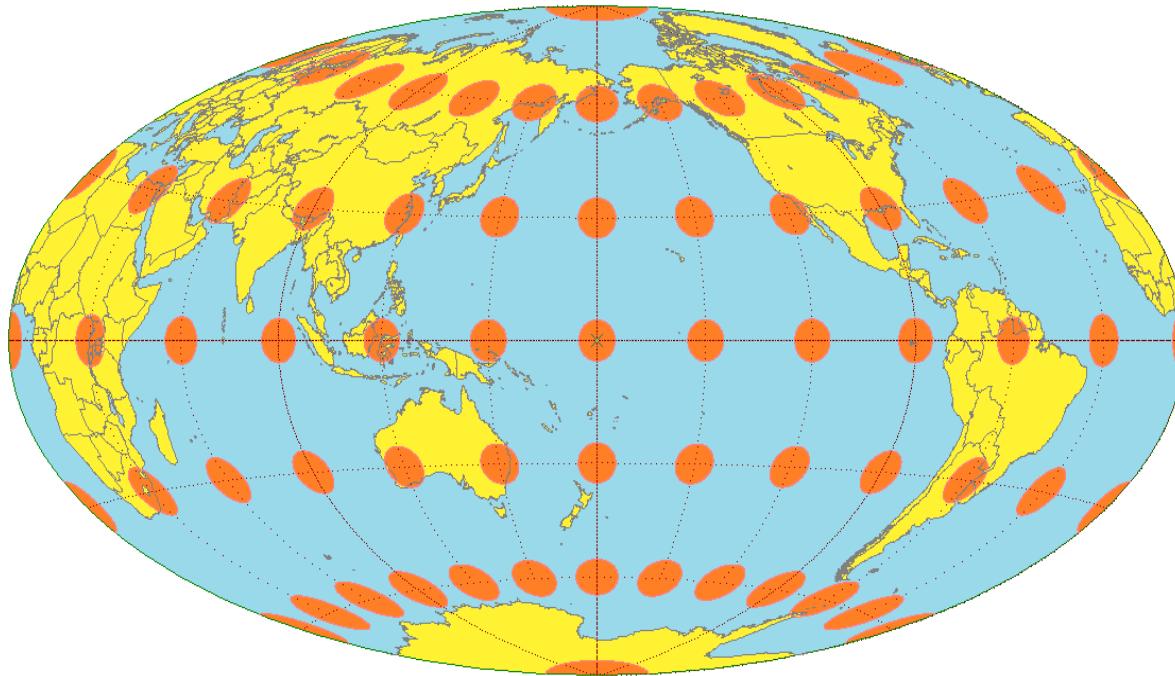
There are many map projections

Most important is the distribution of distortion

How to measure the distortion?

Locally, e.g. by using Tissot's indicatrix or ellipse of distortion

Let  $a$  and  $b$  be semiaxes of such an ellipse



Hammer projection with Tissot's indicatrices showing the distribution of linear distortions.

Source:

[https://commons.wikimedia.org/wiki/File:Hammer\\_projection\\_with\\_Tissot%27s\\_indicatrix.png](https://commons.wikimedia.org/wiki/File:Hammer_projection_with_Tissot%27s_indicatrix.png)

Let us first note that it is always  $a>0$  and  $b>0$   
and that can not be  $a=b=1$  at all points

If  $a=b=1$  at all points then there is a map projection without distortion!

But Euler proved that this was not possible in 1777

At some point or at all points of some line it can be  $a=b=1$

If it is  $a=b=1$  at some point then we say that there is *no distortion* at that point or that the distortion at this point is *equal to zero*

If at all points of a line  $a=1$  then it is not generally the line without distortion, but we can say that this line is *equidistant* in the direction of maximum linear scale

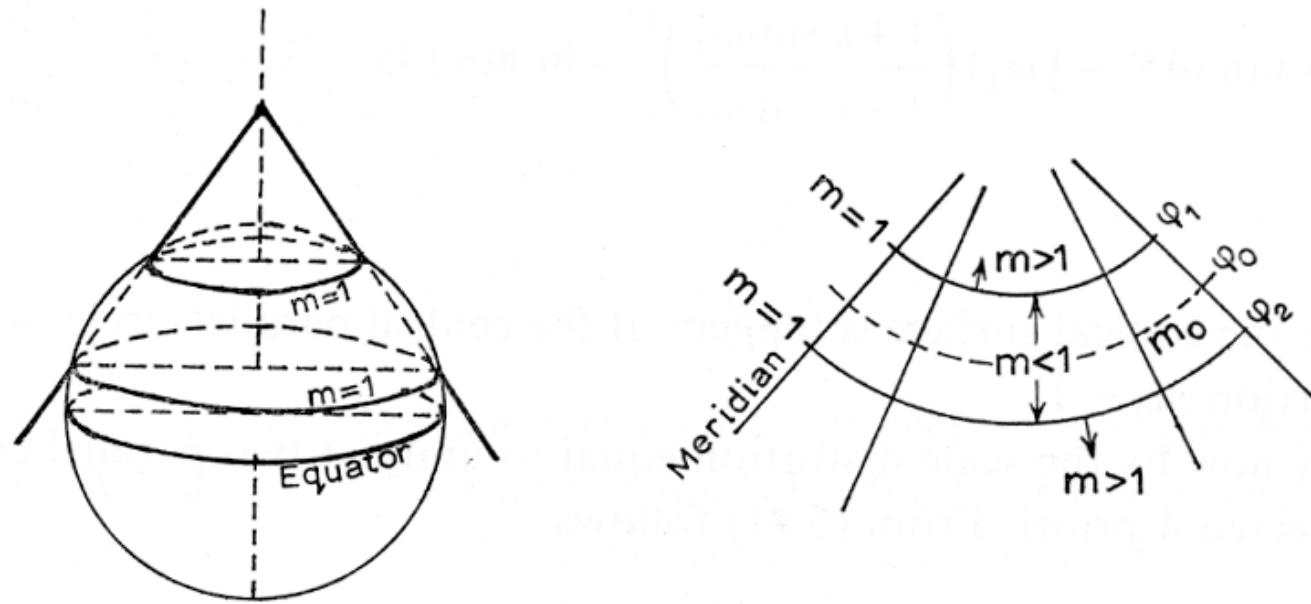
If at all points of a line  $b=1$  then it is not generally the line without distortion, but we can say that this line is *equidistant* in the direction of minimum linear scale

If at all points of a line  $a=b=1$  then it is a *line without distortion*, a *zero-distortion line* or a *standard line*

	$a = 1$	$b = 1$	$a = b = 1$
<b>In a point</b>	11	12	13
<b>Along a line</b>	21	22	23
<b>In an area</b>	31	32	33

Standard points and lines vs. equidistant points and lines

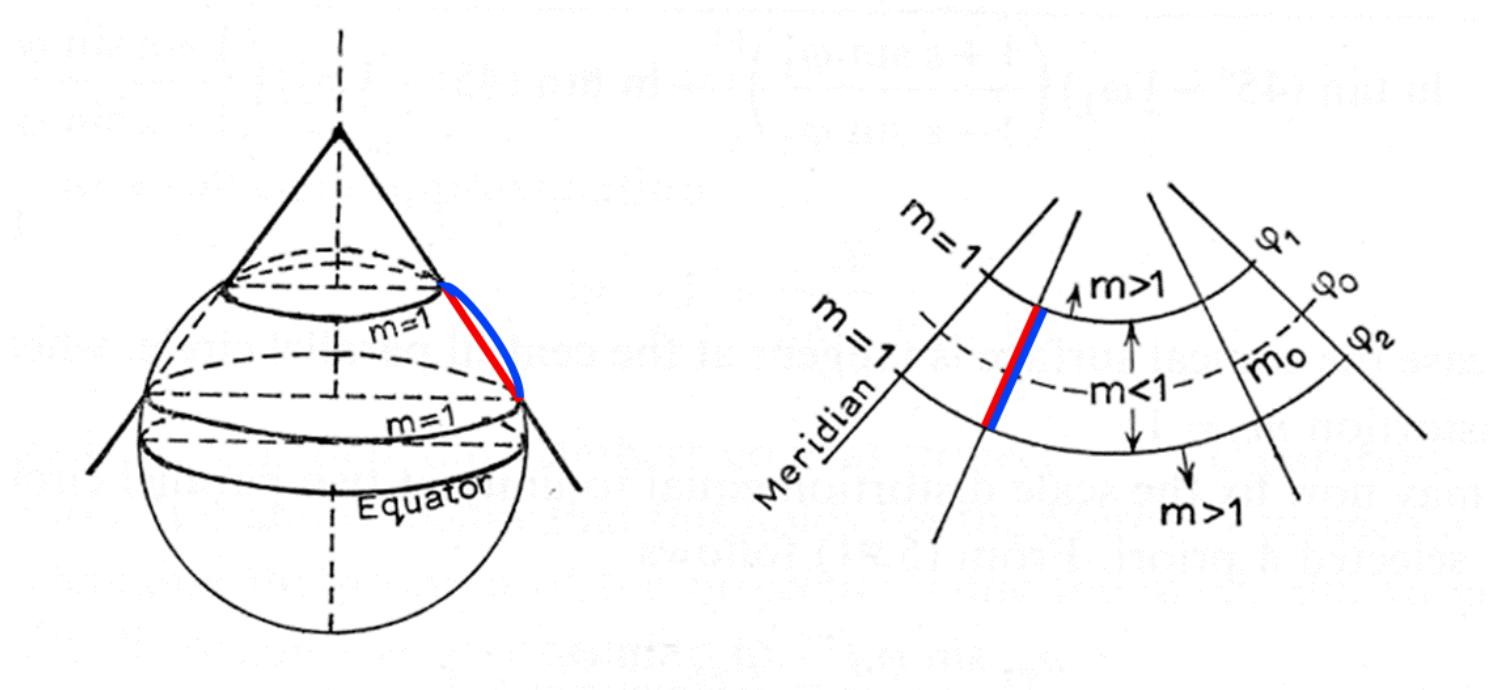
# Illusion or misconception



Source: Richardus and Adler 1972, p. 94.

Generally a misguided approach accompanied by an untruthful statement:

"This is the Lambert conical conformal projection with two standard parallels. The cone intersects the ellipsoid at these parallel circles."



$$n=1$$

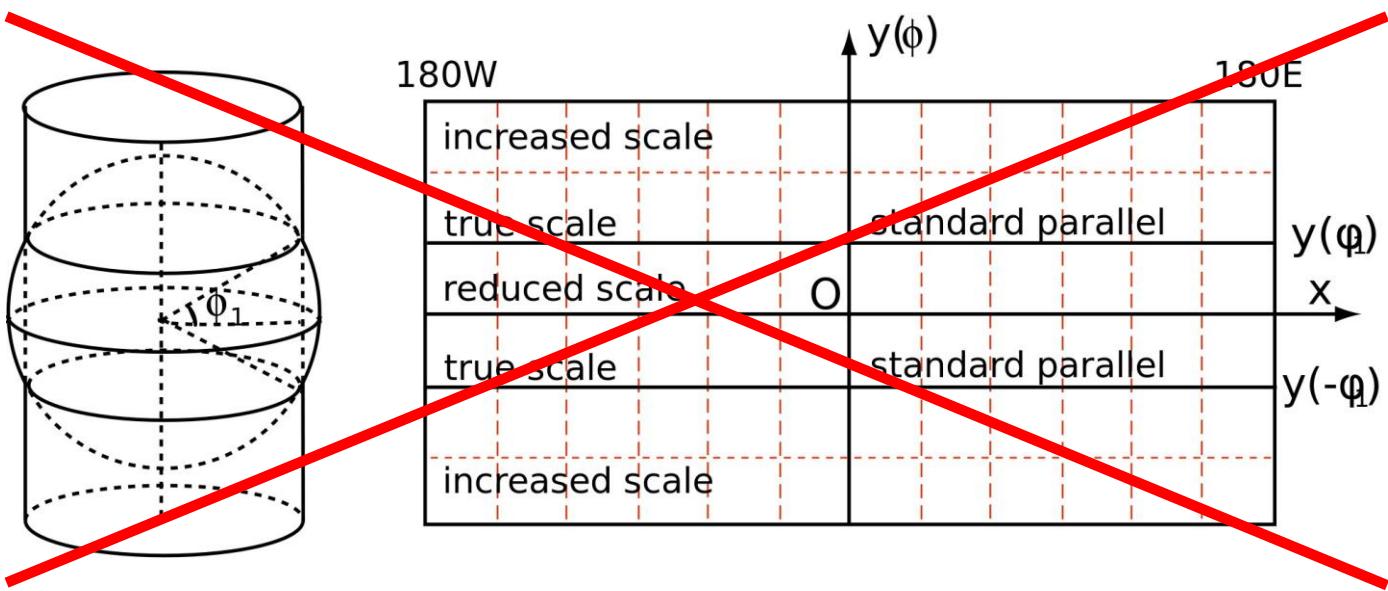
Equidistant projection along meridians

Length of cone generatrix = Arc length of meridian ?

$$mn=1$$

Equal-area projection

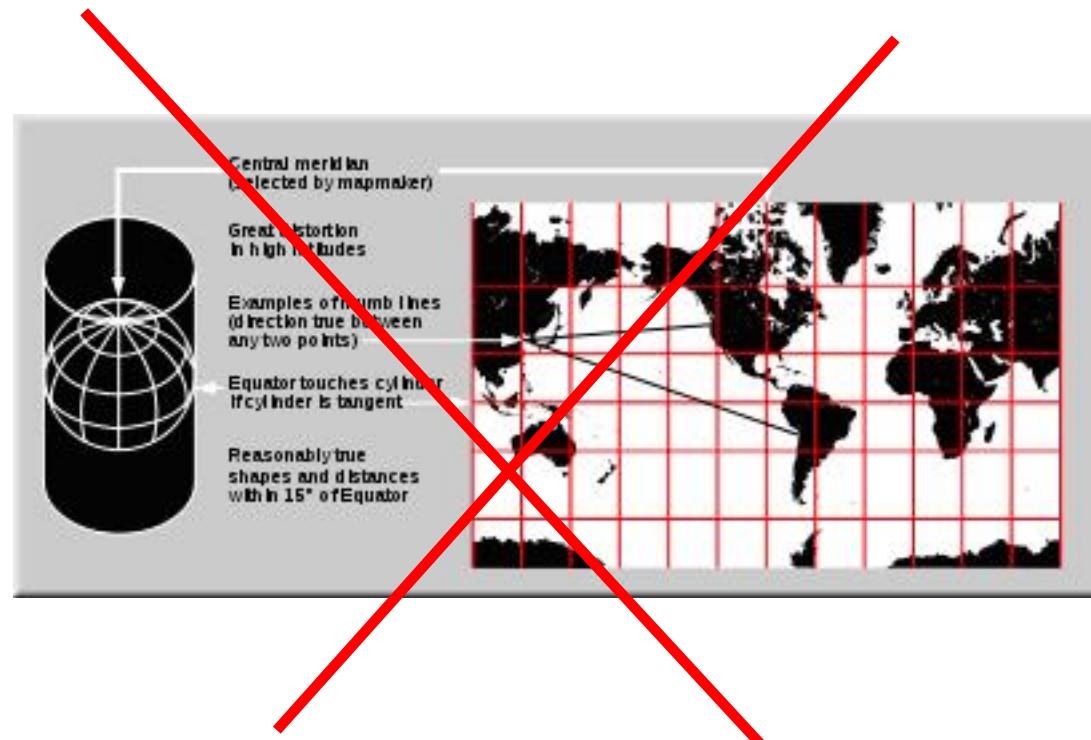
Area of the truncated cone lateral surface = Area of the spherical segment ?

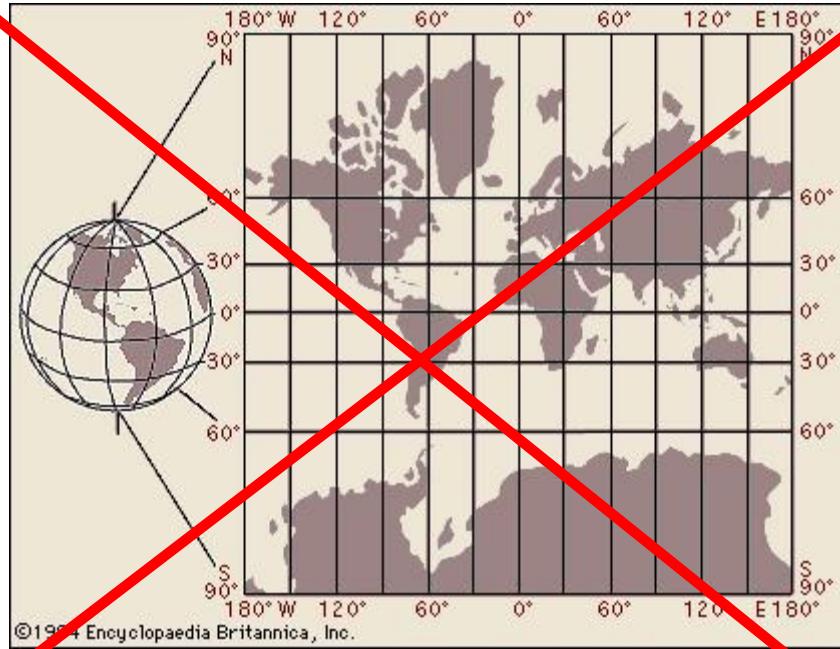




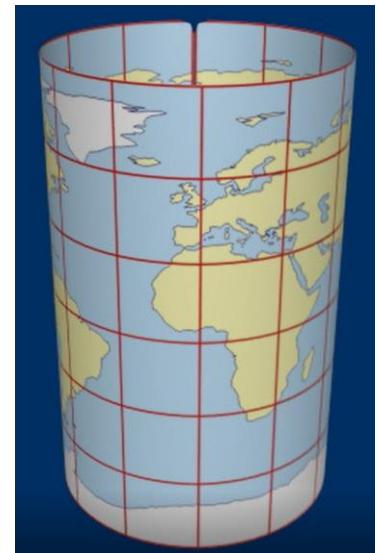
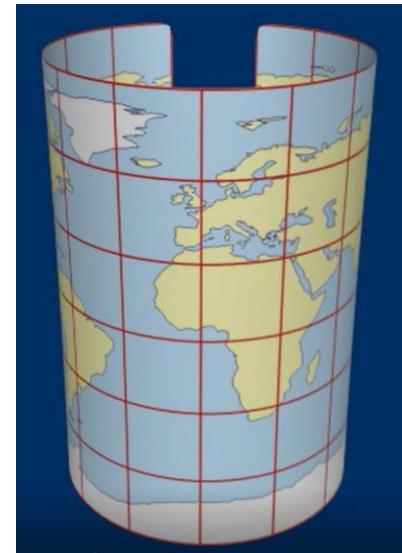
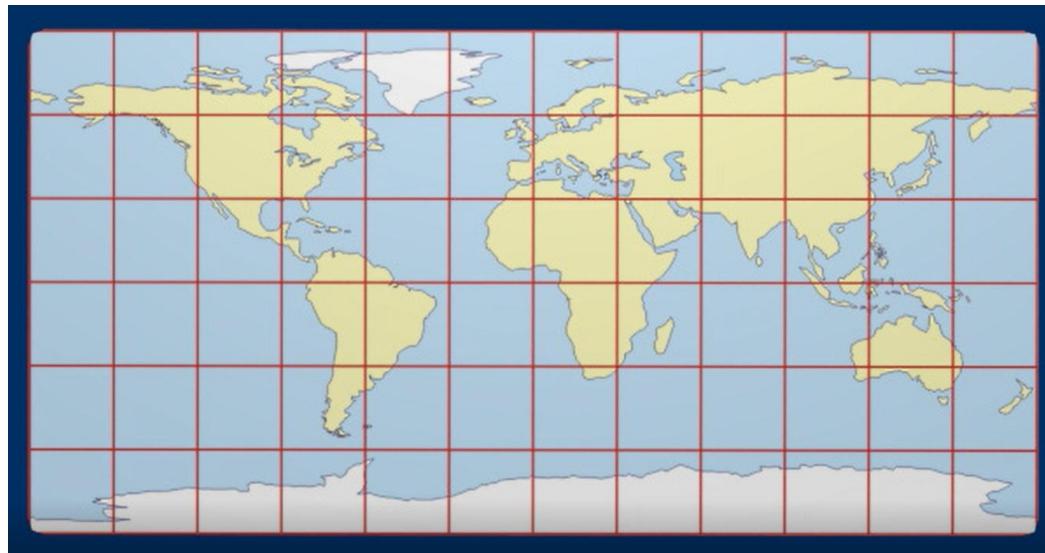
## Let us remember

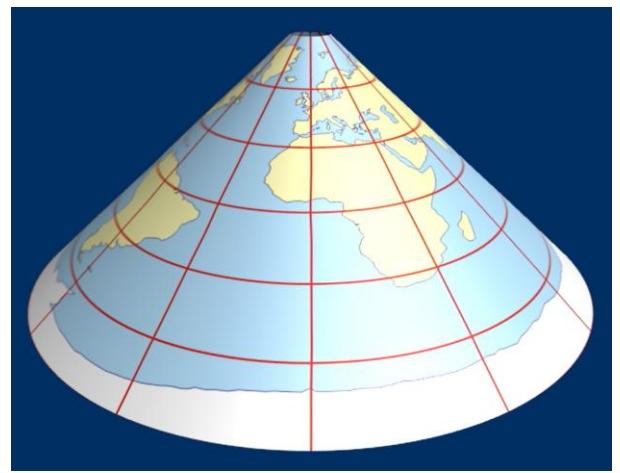
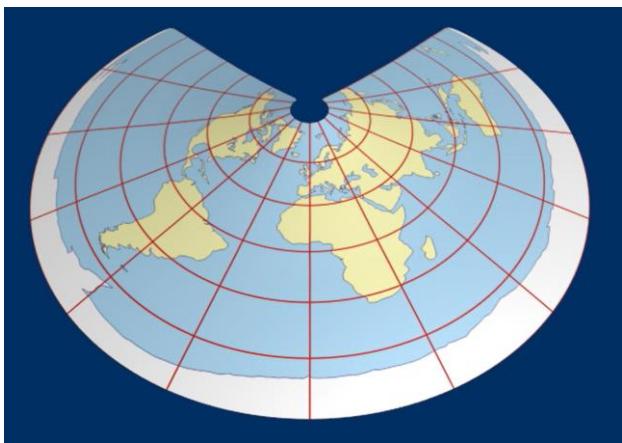
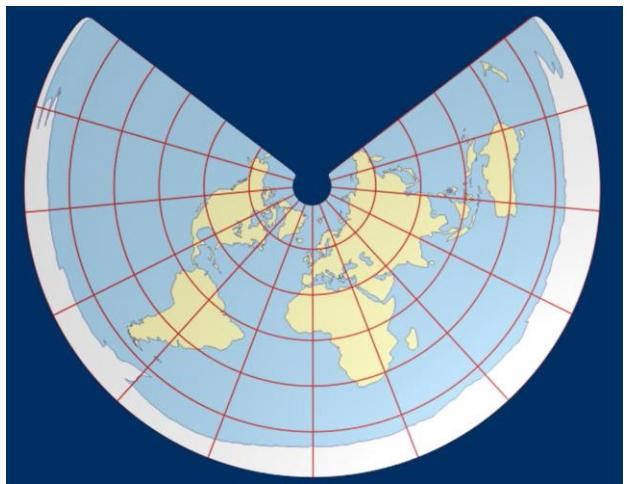
Cylinders do not exist in cylindrical projections in general.

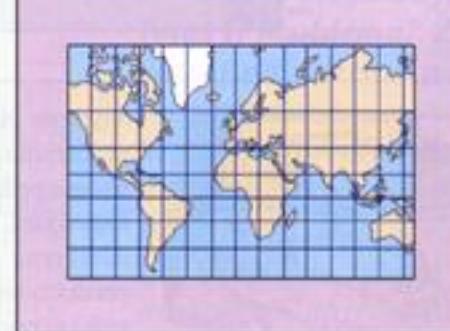
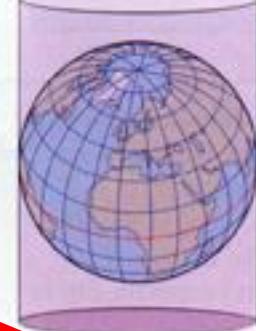




<https://www.britannica.com/science/Mercator-projection>

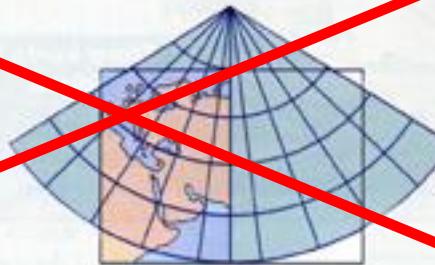






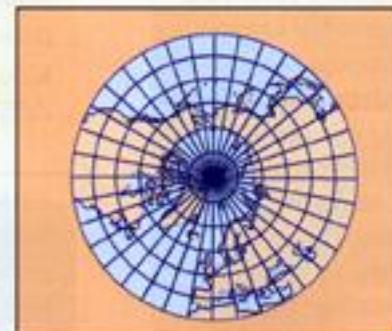
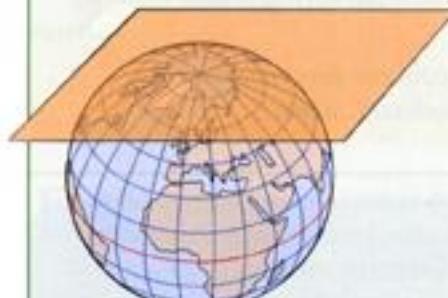
### Valjkasta projekcija

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### Stožasta projekcija

Stožac dodiruje globus na jednoj paraleli. Tu su površine najtočnije prikazane. Dijelovi stožca koji su više odvojeni od globusa, više su i izobličeni.



### Horizontalna projekcija

Ravna ploha dodiruje globus u jednoj točki. Što su površine udaljenije od te središnje točke, to su izobličenije.

# Do we need developable surfaces?

The authors of the oldest projections did not define their projections using auxiliary or developable surfaces



Mercator



Lambert

# Do we need developable surfaces?

Developable surfaces are widely accepted in teaching of map projections.

It is almost impossible to find a publication that deals with map projections in general and without developable surfaces story.

If found, it usually classifies projections as cylindrical, conical and azimuthal/planar, and applies developable surfaces to define the projection aspect.



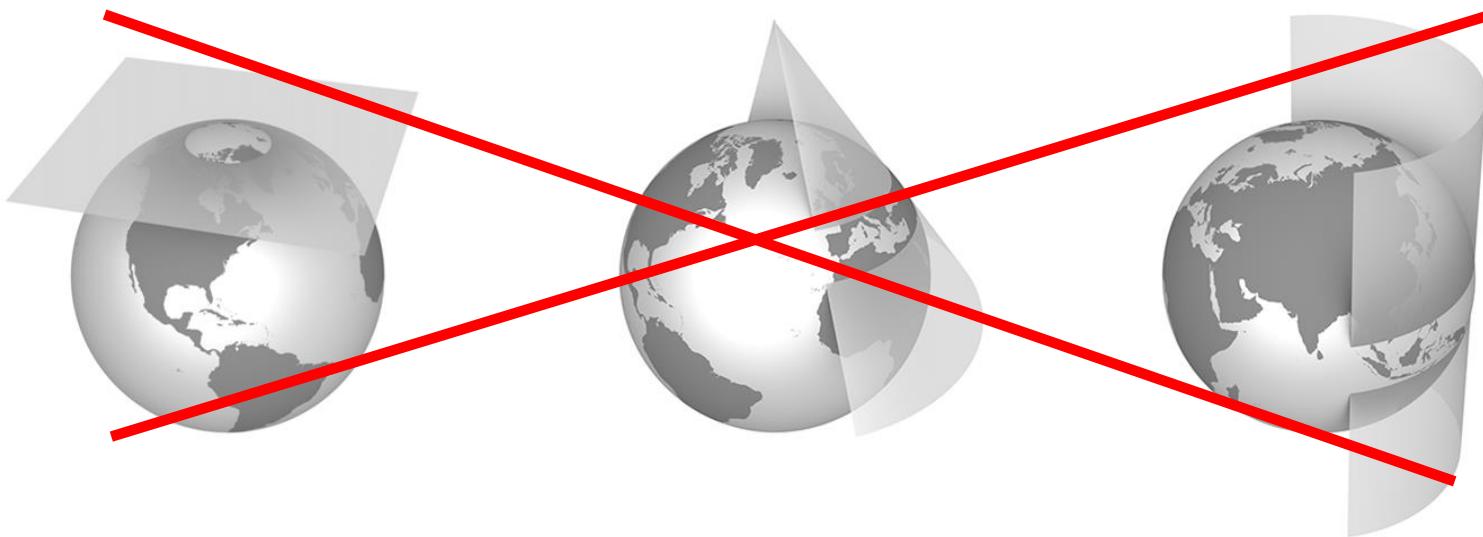
# We don't need developable surfaces

## Let us remember

It is not wise to use auxiliary surfaces when interpreting map projections because:

- Most map projections do not have an auxiliary surface in their definition
- Application of the auxiliary surface can lead to the wrong conclusion about the distribution of deformations (standard parallels)

# Conclusion



Critical thinking is an indispensable condition!