Standard, Equidistant and Secant Parallels

or lack of critical thinking
or about an illusion in map projections

Miljenko Lapaine
The aim of this presentation:

To show some problems we encountered and

The need to continue the fight against illusions in the theory of map projections
Globe is the most faithful representation of the Earth
Map projections and maps are ubiquitous

- atlases
- textbooks
- encyclopedias
- Internet
- …
Pandemic in cartography
Cylindrical projections

http://www2.geof.unizg.hr/~nvucetic/OGI_kart_proj.pdf
Cylindrical projections

http://www.pfri.uniri.hr/~brcic/downloads/3.%20TERESTRICKA%20NAVIGAC
IJA%20Kartografske%20projekcije.pdf
Cylindrical projections

https://hr.wikipedia.org/wiki/Mercatorova_projekcija
Cylindrical projections

https://en.wikipedia.org/wiki/Map_projection
Cylindrical projections

Cylindrical projections

http://www.slideshare.net/yourmohsin/projections-and-coordinate-system
Cylindrical projections

http://lazarus.elte.hu/~guszlev/vet/cylin.htm
Cylindrical projections

Projected plane touches earth surface along one circle

Projected plane touches earth surface along two circles

https://mgimond.github.io/Spatial/coordinate-systems.html
Cylindrical projections

http://support.esri.com/other-resources/gis-dictionary/term/cylindrical%20projection
Cylindrical projections

http://www.codeguru.com/cpp/g-m/bitmap/viewers/article.php/c9187/2D--3D-Visualization-Techniques-for-GeoReferenced-Images.htm
Cylindrical projections

http://nptel.ac.in/courses/105102015/42
Cylindrical projections

http://maps.unomaha.edu/Peterson/gis/notes/MapProjCoord.html
Cylindrical projections

http://www.geography.hunter.cuny.edu/~jochen/GTECH361/lectures/lecture04/concepts/Map%20coordinate%20systems/Cylindrical%20projections.htm
Cylindrical projections

http://web.gccaz.edu/~lnewman/gph111/topic_units/systems_grid_proj/systems_time/systems_time2.html
The Mercator projection—a transformation from the simple cylindrical projection—is used for navigation, since lines of constant direction on the Earth appear as straight lines on the map.

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• **Gauss-Krüger projekcija**
  • Austrija je još 1917. uvela za potrebe državne izmjere, kasnije Njemačka i bivša Jugoslavija

• **Konformna poprečna cilindrična projekcija**
• **Elipsoid – Bessel 1841**

• Dijeli se na zone široke 3° unutar kojih su deformacije male
• Svaka zona ima drugi dodirni meridian
  • 15° /3° = 5° => zona 5
  • 18° /3° = 6° => zona 6
• U svakoj zoni je projekcija središnjeg meridijana je **os x koordinatnog sustava, a os y je projekcija ekvatoria**
• y koordinata se uvećava za 500 000 m zbog izbjegavanja negativnih vrijednosti
Valjasta projekcija
Valjak dodiruje globus na ekvatoru. Tamo su površine najvjernije, a na polovima su najizobilješnije.

Stožasta projekcija
Stožac dodiruje globus na jednoj paraleli. Tu su površine najtočnije prikazane. Dijelovi stošca koji su više odvojeni od globusa, više su i izobilješnji.

Horizontalna projekcija
Ravna ploha dodiruje globus u jednoj točki. Što su površine udaljenije od te središnje točke, to su izobilješnije.
Prof. Dr. at the Faculty of Electrical Engineering and Computing:

In addition to the cylindrical surface, cone, pseudo and sinusoidal cylindrical surfaces and planes can be used for projection.
Pseudosomething ...
It is obviously a pandemic
For information on indications, precautions and side effects ask your doctor or pharmacist
Contributions to the Use of Mathematics and its Applications
VERÖFFENTLICHUNG
DES KÖNIGLICH PREUSZISCHEN GEODÄTISCHEN INSTITUTES
NEUER FOLGEN 28

KONFORME ABBILDUNG
DES ERDPELLSOIDS IN DER EBENE

VON

Prof. Dr. L. KRÜGER

ANTHROPOMETRIE IM KAISERLICHEREN GEODÄTISCHEN INSTITUT

POTS DAM

DRUCK UND VERLAG VON B. G. TEUBNER IN LEIPZIG

1912
Conical projections are those in which the parallels are represented by concentric circles and the meridians by equally spaced radii. There is no necessary connexion between a conical projection and any touching or secant cone.

The name conical is given to the group embraced by the above definition, because, as is obvious, a projection so drawn can be round to form a cone.

*Cylindric*: projections in which the meridians are represented as a system of equidistant parallel straight lines, and the parallels by a system of parallel straight lines at right angles to the meridians.

*Conic*: projections in which the meridians are represented as ...

*Azimuthal*: projections in which the meridians are represented as ...

No cylinders, no cones, ... ?!
"No reference has been made in the above definitions to cylinders, cones or planes. The projections are termed cylindric or conic because they can be regarded as developed on a cylinder or cone, as the case may be, but it is as well to dispense with picturing cylinders and cones, since they have given rise to much misunderstanding.

Particularly is this so with regards to the conic projections with two standard parallels: they may be regarded as developed on cones, but they are cones which bear no simple relationship to the sphere."
There are many map projections

Most important is the distribution of distortion

How to measure the distortion?

Locally, e.g. by using Tissot’s indicatrix or ellipse of distortion

Let $a$ and $b$ be semiaxes of such an ellipse
Hammer projection with Tissot's indicatrices showing the distribution of linear distortions.
Source: https://commons.wikimedia.org/wiki/File:Hammer_projection_with_Tissot_%27s_indicatrix.png
Let us first note that it is always $a>0$ and $b>0$ and that can not be $a=b=1$ at all points.

If $a=b=1$ at all points then there is a map projection without distortion!

But Euler proved that this was not possible in 1777.
At some point or at all points of some line it can be $a=b=1$

If it is $a=b=1$ at some point then we say that there is *no distortion* at that point or that the distortion at this point is *equal to zero*

If at all points of a line $a=1$ then it is not generally the line without distortion, but we can say that this line is *equidistant* in the direction of maximum linear scale

If at all points of a line $b=1$ then it is not generally the line without distortion, but we can say that this line is *equidistant* in the direction of minimum linear scale

If at all points of a line $a=b=1$ then it is a *line without distortion*, a *zero-distortion line* or a *standard line*
<table>
<thead>
<tr>
<th></th>
<th>(a = 1)</th>
<th>(b = 1)</th>
<th>(a = b = 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>In a point</strong></td>
<td>11</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td><strong>Along a line</strong></td>
<td>21</td>
<td>22</td>
<td>23</td>
</tr>
<tr>
<td><strong>In an area</strong></td>
<td>31</td>
<td>32</td>
<td>33</td>
</tr>
</tbody>
</table>

Standard points and lines vs. equidistant points and lines
Generally a misguided approach accompanied by an untruthful statement: "This is the Lambert conical conformal projection with two standard parallels. The cone intersects the ellipsoid at these parallel circles."

Source: Richardus and Adler 1972, p. 94.
n=1
Equidistant projection along meridians
Length of cone generatrix = Arc length of meridian

mn=1
Equal-area projection
Area of the truncated cone lateral surface = Area of the spherical segment
<table>
<thead>
<tr>
<th>Scale Type</th>
<th>Standard Parallel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Increased Scale</td>
<td></td>
</tr>
<tr>
<td>True Scale</td>
<td>O</td>
</tr>
<tr>
<td>Reduced Scale</td>
<td></td>
</tr>
<tr>
<td>Increased Scale</td>
<td></td>
</tr>
</tbody>
</table>
Let us remember

Cylinders do not exist in cylindrical projections in general.
https://www.britannica.com/science/Mercator-projection
Valjak dodiruje globus na ekvatoru. Tamo su površine najjednako, a na polovima su najizobiljene.

Stožasti projekt dijagram

Stožac dodiruje globus na jednoj paraleli. Tu su površine najtočnije prikazane. Dijelovi stošca koji su više odvojeni od globusa više su i izobiljene.

Horizontalna projekcija

Ravna ploha dodiruje globus u jednoj točki. što su površine udaljene od te središnje točke, to su izobiljene.
Do we need developable surfaces?

The authors of the oldest projections did not define their projections using auxiliary or developable surfaces.
Do we need developable surfaces?

Developable surfaces are widely accepted in teaching of map projections.

It is almost impossible to find a publication that deals with map projections in general and without developable surfaces story.

If found, it usually classifies projections as cylindrical, conical and azimuthal/planar, and applies developable surfaces to define the projection aspect.
We don’t need developable surfaces

Let us remember

It is not wise to use auxiliary surfaces when interpreting map projections because:

• Most map projections do not have an auxiliary surface in their definition

• Application of the auxiliary surface can lead to the wrong conclusion about the distribution of deformations (standard parallels)
Critical thinking is an indispensable condition!