EVIDENCE OF PORTOLAN CHART
LOXODROMIC GEOMETRY
the Example of the Adriatic Sea Basin

Tome Marelić
University of Zadar
Department of Geography
tmarelic@unizd.hr
THE RIDDLE OF PORTOLAN CHARTS’ GEOMETRY

▶ Earliest known appearance – end of 13th century (Pisan Chart from 1290 or possibly even earlier).

▶ **Very accurate geometry** – in terms of mapping accuracy portolan charts are incomparably superior to their earliest known map predecessors, known as mappae mundi.

▶ **Appeared suddenly in almost fully developed form** since the earliest known charts.

▶ Explicit progress (evolution) in mapping accuracy that links portolan charts to their closest known map predecessors is NOT PRESERVED in historical sources, nor are the methods of their construction, and methods of prerequisite (geodetic) survey.
THE RIDDLE OF PORTOLAN CHARTS’ GEOMETRY

Close resemblance to geometric properties of equidistant cylindrical projection and Mercator projection (Wagner, 1896; Clos-Arceduc, 1956; Tobler, 1966; Loomer, 1987; Nicolai, 2014) – especially when examined as composites made of standalone sea-basin charts.

Geometries of portolan charts shows relatively high level of standardization: CA 1375 – Adriatic Sea in ‘Catalan Atlas’ (1375), DHa 1570 – Adriatic Sea in atlas of Diogo Homem (1570)
RESEARCH GOALS

- To determine fundamental geometric properties of portolan charts using (digital) cartometric analysis.
- To test geometric correlation between geometries of portolan charts and geometries of (selected) map projections.
- To test possibility of loxodromic properties of portolan charts.

RESEARCH OBJECT / AREA

- Renderings of Adriatic Sea basin coastline on selected portolan charts.
**SELECTED PORTOLAN CHARTS**

<table>
<thead>
<tr>
<th>ABBREVIATION</th>
<th>DESCRIPTION</th>
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<tbody>
<tr>
<td><strong>CP 1290</strong></td>
<td><em>Pisan Chart</em> (of unknown author, or group of authors) (1290)</td>
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<tr>
<td><strong>PV 1318</strong></td>
<td>chart of Adriatic Sea from <em>Pietro Vesconte’s atlas</em> (1318)</td>
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<tr>
<td><strong>CA 1375</strong></td>
<td>chart of central Mediterranean from <em>Catalan Atlas</em> (1375) whose authorship is assigned to Abraham Cresques</td>
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<tr>
<td><strong>AB 1436</strong></td>
<td>chart of central and western Mediterranean from <em>Andrea Bianco’s atlas</em> (1436)</td>
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<tr>
<td><strong>GB 1466</strong></td>
<td>chart of central Mediterranean from <em>Gratiosus Benincasa’s atlas</em> (1466)</td>
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<tr>
<td><strong>AN 1482</strong></td>
<td>chart of central Mediterranean from <em>anonymous atlas</em> (1482) which aesthetically closely resembles charts of Gratiosus Benincasa</td>
</tr>
<tr>
<td><strong>BAa 1538 / BAb 1538</strong></td>
<td>(a) chart of central and eastern Mediterranean and SW part of Black Sea, (b) chart of Adriatic Sea – both from <em>Battista Agnese’s atlas</em> (1538)</td>
</tr>
<tr>
<td><strong>DHa 1570 / DHb 1570</strong></td>
<td>(a) chart of central and eastern Mediterranean and SW part of Black Sea, (b) chart of Adriatic Sea – both from <em>Diogo Homem’s atlas</em> (1570)</td>
</tr>
<tr>
<td><strong>VDVa 1593 / VDVb 1593</strong></td>
<td>(a) chart of central and eastern Mediterranean and SW part of Black Sea, (b) chart of Adriatic Sea – both from <em>Vicko Dimitrije Volčić’s atlas</em> (1593)</td>
</tr>
</tbody>
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**METHODOLOGY (CARTOMETRIC ANALYSIS)**

- Geometric comparison of Adriatic Sea basin coast renderings on (12) selected portolan charts with its rendering on a modern (reference) map projected in (6) selected map projections.

- Superimposing *triangulated irregular networks (TINs)* over maps in reference projections (6 *reference* TINs) and over selected portolan charts (12 *identical* TINs).

- Calculation fundamentals: 21 *selected reference-identical point pairs* for geometry comparisons (*LSE* process $\rightarrow$ *RMSE calculations* between reference and identical values).

- Application of planar 4-parameter similarity (Helmert) *transformation* in order to preserve charts’ initial geometry.
Construction of superimposed (reference) TIN that consists of 21 selected points and 43 lines drawn between them.
CONSTRUCTION REQUIREMENTS FOR TINs

▷ All identical points (21) have to be visible and recognizable on all portolan charts.

▷ Outer limits of TINs have to be drawn in a manner that they represent simplified geometry of Adriatic Sea basin.

▷ TIN lines have to be drawn in a way that graphic collision with parts of the map/chart that represent land is minimal. In other words, lines have to represent paths (courses) that are fully navigable or almost fully navigable (especially the cross-basin ones).
Construction of superimposed (identical) TINs over selected portolan charts (2 out of 12 charts are shown)
Evidence of Portolan Chart Loxodromic Geometry (the Example of the Adriatic Sea Basin)

Georeferenced portolan chart with an application of planar 4-parameter conformal transformation

Chart is considered as being georeferenced after displacements of identical points (compared to their reference equivalents) are minimal.

To obtain that, Least Squares Estimation (LSE) has to be calculated first in order to extract minimal possible RMSE value(s) between the two shapes (maps in this case).

LSE process → Georeferencing → RMSE calculations

Mercator projection $\phi_c$=42.7°
Evidence of Portolan Chart Loxodromic Geometry (the Example of the Adriatic Sea Basin)
Evidence of Portolan Chart Loxodromic Geometry (the Example of the Adriatic Sea Basin)
Renderings of Adriatic Sea basin coastline on a map differ in dependence of (reference) map projection

On oblique gnomonic projection and on HTRS96/TM convergence of meridians is shown, while on other (cylindrical) projections meridians are rendered as straight parallel lines.

In terms of map scale, projections 1, 2, 3 and 5 can be considered as local projections, while projections 4 and 6 can be considered as regional projections.

SELECTED REFERENCE MAP PROJECTIONS*:
1. HTRS96/TM
2. oblique gnomonic $\varphi_o=+42.7^\circ \lambda_o=+16^\circ$
3. Mercator $\varphi_o=42.7^\circ$
4. Mercator $\varphi_o=36^\circ$
5. equidistant cylindrical $\varphi_o=42.7^\circ$
6. equidistant cylindrical $\varphi_o=36^\circ$

*coastline renderings on all selected projections are displayed in same scale factor.

Adriatic Sea basin coastline rendering:
- within research area
- outside research area

Evidence of Portolan Chart Loxodromic Geometry (the Example of the Adriatic Sea Basin)
part 1: THE MOST SIMILAR MAP PROJECTION

- POSITION MAPPING ACCURACY:
  - Along X-axis – $\text{RMSE } dX$ [km]
  - Along Y-axis – $\text{RMSE } dY$ [km]

- DISTANCE MAPPING ACCURACY:
  - $\text{RMSE } dL$ [km]

- BEARING MAPPING ACCURACY:
  - $\text{RMSE } d\alpha$ [$^\circ$]
EVIDENCE OF PORTOLAN CHART LOXODROMIC GEOMETRY (THE EXAMPLE OF THE ADRIATIC SEA BASIN)

POSITION MAPPING ACCURACY ($RMSE_{dX}$)

$$RMSE_{dX} = \sqrt{\frac{\sum_{i=1}^{n} (X_i - \hat{X}_i)^2}{2n - 4}}$$

HIGHEST ACCURACY:
- **Mercator** $\phi_0=42.7^\circ$
- **Eqdist. Cyl.** $\phi_0=42.7^\circ$

AVG $RMSE_{dX} = \text{10.1 km}$

LOWEST ACCURACY:
- **Oblique Gnomonic**
- **HTRS96/TM**

AVG $RMSE_{dX} = \text{12.1 km}$

- **HTRS96/TM A**
- **HTRS96/TM B**
- **Gn. $\Phi_0=42.7^\circ$ $\lambda_0=+16^\circ$ A**
- **Gn. $\Phi_0=42.7^\circ$ $\lambda_0=+16^\circ$ B**
- **Merc. $\Phi_0=42.7^\circ$ A**
- **Merc. $\Phi_0=42.7^\circ$ B**
- **Merc. $\Phi_0=36^\circ$ A**
- **Merc. $\Phi_0=36^\circ$ B**
- **Eqdist. Cyl. $\Phi_0=42.7^\circ$ A**
- **Eqdist. Cyl. $\Phi_0=42.7^\circ$ B**
- **Eqdist. Cyl. $\Phi_0=36^\circ$ A**
- **Eqdist. Cyl. $\Phi_0=36^\circ$ B**
POSITION MAPPING ACCURACY \( (RMSE \, dY) \)

\[
RMSE \, dY = \sqrt{\frac{\sum_{i=1}^{n}(Y_i - \bar{Y}_i)^2}{2n - 4}}
\]

**HIGHEST ACCURACY:**
- **Mercator** \( \varphi_0=42.7^\circ \)
  - AVG RMSE \( dY = 12.7 \text{ km} \)

**LOWEST ACCURACY:**
- **Equidist. Cyl.** \( \varphi_0=36^\circ \)
  - AVG RMSE \( dY = 15.9 \text{ km} \)

**Evidence of Portolan Chart Loxodromic Geometry (the Example of the Adriatic Sea Basin)\)**
**DISTANCE MAPPING ACCURACY (RMSE dL)**

\[
RMSE_{dL} = \sqrt{\frac{\sum_{i=1}^{n}(L_i - \hat{L}_i)^2}{n - 1}}
\]

**HIGHEST ACCURACY:**

- **Mercator \( \phi_0=42,7^\circ \)**
  
  AVG RMSE dL = 22.1 km

**LOWEST ACCURACY:**

- **Mercator \( \phi_0=36^\circ \)**
  
  AVG RMSE dL = 24.3 km

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Evidence of Portolan Chart Loxodromic Geometry (the Example of the Adriatic Sea Basin)
**BEARING MAPPING ACCURACY** ($RMSE \, d\alpha$)

\[
RMSE \, d\alpha = \sqrt{\frac{\sum_{i=1}^{n}(\alpha_i - \hat{\alpha}_i)^2}{n-1}}
\]

**HIGHEST ACCURACY:**
- **Mercator (in general)**
  AVG $RMSE \, d\alpha = 8.92^\circ$

**LOWEST ACCURACY:**
- **Equidist. Cyl. $\varphi_o=36^\circ**
  AVG $RMSE \, d\alpha = 9.56^\circ$

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Evidence of Portolan Chart Loxodromic Geometry (the Example of the Adriatic Sea Basin)
CONCLUSIONS (THE MOST SIMILAR MAP PROJECTION)

▷ If, hypothetically, graticules were to be drawn on portolan charts, the convergence of meridians would not be shown, that is, meridians would be drawn as straight parallel lines (lowest RMSE dX accuracy in comparison with map projections that show convergence of meridians).

▷ Geometric properties of renderings of Adriatic Sea basin coastline on portolan charts are most similar to those of Mercator map projection (Mercator projection $\varphi_0=42.7^\circ$ is selected as a reference map projection for the purpose of following analysis).
REGIONAL DIFFERENCES IN MAPPING ACCURACY

- **DISTANCE MAPPING ACCURACY:**
  - ABSOLUTE VALUES – $MAE \, dl \,[km]$  
  - BIASED VALUES – $ME \, dl \,[km]$  

- **BEARING MAPPING ACCURACY:**
  - ABSOLUTE VALUES – $MAE \, d\alpha \,[^\circ]$
ABSOLUTE DISTANCE MAPPING ACCURACY (MAE dL)

\[ MAE \, dL = \frac{\sum_{i=1}^{n} |L_i - \hat{L}_i|}{n - 1} \]

Absolute (unbiased) distance mapping errors in comparison with Mercator \( \phi_0 = 42.7\degree \) as a reference map projection

AVG MAE dL (reg. division):

- north Adriatic A: 31.5 km
- central Adriatic A: 14.6 km
- south Adriatic A: 16.3 km
- whole Adriatic A: 17.6 km

Evidence of Portolan Chart Loxodromic Geometry (the Example of the Adriatic Sea Basin)
Spatial distribution of absolute distance mapping errors, |dL| [km] in comparison with Mercator $\varphi_0=42.7^\circ$ as a reference map projection.
Evidence of Portolan Chart Loxodromic Geometry (the Example of the Adriatic Sea Basin)

**BIASED DISTANCE MAPPING ACCURACY** \((ME \, dL)\)

\[
ME \, dL = \frac{\sum_{i=1}^{n}(L_i - \hat{L}_i)}{n - 1}
\]

Biased distance mapping errors in comparison with Mercator \(\varphi_0=42.7^\circ\) as a reference map projection

**AVG ME dL (reg. division):**
- north Adriatic: +26.9 km
- central Adriatic: +5.0 km
- south Adriatic: +1.6 km
- whole Adriatic: +7.9 km
Evidence of Portolan Chart Loxodromic Geometry (the Example of the Adriatic Sea Basin)

Spatial distribution of biased distance-mapping errors, $dL$ [km] in comparison with Mercator $\phi_0 = 42.7^\circ$ as a reference map projection.
ABSOLUTE BEARING MAPPING ACCURACY \((\text{MAE } d\alpha)\)

\[
\text{MAE } d\alpha = \frac{\sum_{i=1}^{n} |\alpha_i - \hat{\alpha}_i|}{n - 1}
\]

Absolute (unbiased) bearing mapping errors in comparison with Mercator \(\phi_0=42.7^\circ\) as a reference map projection

**AVG MAE \(d\alpha\) (reg. division):**
- **north Adriatic A:** 6.61°
- **central Adriatic A:** 8.77°
- **south Adriatic A:** 5.76°
- **whole Adriatic A:** 7.01°

Evidence of Portolan Chart Loxodromic Geometry (the Example of the Adriatic Sea Basin)
UNBIASED BEARING MAPPING ACCURACY (MAE $d\alpha$)

$MAE \, d\alpha = \frac{\sum_{i=1}^{n} |\alpha_i - \hat{\alpha}_i|}{n-1}$

Absolute (unbiased) bearing mapping errors in comparison with Mercator $\phi_0=42.7^\circ$ as a reference map projection

AVG $MAE \, d\alpha$ (reg. division):
- coastal lines: 6.86°
- cross-basin lines: 5.67°
- island lines: 9.06°
- whole Adriatic: 7.01°

Evidence of Portolan Chart Loxodromic Geometry (the Example of the Adriatic Sea Basin)
Spatial distribution of absolute bearing mapping errors $|d\alpha|$ [$^\circ$] in comparison to Mercator $\varphi_0 = 42.7^\circ$ as a reference map projection.
CONCLUSIONS
(REGIONAL DIFFERENCES IN MAPPING ACCURACY)

- Cross-basin lines on portolan charts show relatively low and standardized bearing error values for both north and south part of Adriatic Sea basin, while distance errors increase incrementally in a south-north direction. This pattern of mapping distortions is similar to that of the Mercator projection.

- Renderings of Adriatic Sea basin coastline on portolan charts seem to be a product of defined and intentionally applied procedure of conformal (loxodromic) projecting of (Earth as a) sphere into plane.
BIBLIOGRAPHY


The research results presented here are part (2 chapters out of 7) of the author's broader ongoing doctoral thesis research, entitled: *The Accuracy of Depictions of the Adriatic Sea on Portolan Charts.*

If there is any additional interest in results presented here (or in the rest of the research), feel free to contact me at: tmarelic@unizd.hr

THANK YOU FOR YOUR ATTENTION!